Online Appendix to “A Theory of Strategic Uncertainty and Cultural Diversity”

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I Strategic uncertainty

In the main text, we claimed that culturally diverse societies face more strategic uncertainty than culturally homogeneous societies. Here we make that claim precise. Strategic uncertainty is high if players are not very informative, i.e., if posterior beliefs are close to the prior (cf. Figure 1). In the current setting, this is equivalent to the variance in impulses being high.

We calculate the variance in impulses and show that the variance in impulses is higher in diverse societies. Fix $Q_{in}, Q_{out}$ and recall that $Q_{in}, Q_{out}$ are functions of culture strength $q$ and cultural distance $d$. Suppose the level of diversity is $\beta$ so that the minority and majority shares are $\beta$ and $\tilde{\beta} = 1 - \alpha$, respectively. For a player $j \in N$ who belongs to the minority group, the expected proportion of players who have the same impulse as he does is $Q_{min}(\beta; q, d) := \tilde{\beta}Q_{out} + \beta Q_{in}$. Likewise, for a player $j$ who belongs to the majority group, the expected proportion of players who have the same impulse as he does is $Q_{maj}(\beta; q, d) := \tilde{\beta}Q_{in} + \beta Q_{out}$. Since $\tilde{\beta} \geq \frac{1}{2}$ and $Q_{in} > Q_{out} > \frac{1}{2}$, $Q_{maj}(\beta; q, d) \geq Q_{min}(\beta; q, d) > \frac{1}{2}$ (with strict inequality if $\beta < \frac{1}{2}$). Then, the degree of strategic uncertainty that a player in the minority and the majority face is given by

$$V_{min}(\beta; q, d) := Q_{min}(\beta; q, d) (1 - Q_{min}(\beta; q, d)),$$

$$V_{maj}(\beta; q, d) := Q_{maj}(\beta; q, d) (1 - Q_{maj}(\beta; q, d)),$$

respectively. So, the majority faces less strategic uncertainty than the minority (i.e., $V_{maj}(\beta; q, d) < V_{min}(\beta; q, d)$). We can also define aggregate strategic uncertainty $V$ for the society by

$$V(\beta; q, d) := \tilde{\beta}V_{maj}(\beta; q, d) + \beta V_{min}(\beta; q, d).$$

As $V(\beta; q, d)$ is increasing in diversity $\beta$, players face less strategic uncertainty in culturally homogeneous societies than in culturally diverse ones. Also, there is less uncertainty in societies with a strong culture ($V(\beta; q, d)$ decreases with $q$) and that the difference between homogeneous and diverse societies is larger when the culture is strong and when the cultural distance between groups is large. That is, if $\beta' > \beta$, then $V(\beta; q, d) - V(\beta'; q, d)$ increases with $q$ and with $d$. 

1
II Semi-convex games

To illustrate the broader applicability of the present analysis, this appendix extends the results for linear games to a broader class of games. Say that a game is semi-convex if $g(m)$ is strictly increasing and convex in $m$ and satisfies $g(\frac{1}{2}) > (x + \frac{1}{2})g(\frac{1}{2} - x) + (\frac{1}{2} - x)g(x + \frac{1}{2})$ for $x \in (0, \frac{1}{2})$. This inequality implies that $g(m)$ cannot be too convex in the sense that $g(m)$ cannot be nearly flat for $m < \frac{1}{2}$ and increase steeply for $m > \frac{1}{2}$. Semi-convex games include linear games, but also include other games, as we illustrate below. We restrict attention to semi-convex games with identical preferences ($\rho + j = \rho$ for all $j \in N$) for simplicity.

The first result extends Proposition A.2 to semi-convex games:

Proposition II.1. [Introspective Equilibrium] For any semi-convex game with identical preferences:

(a) An introspective equilibrium exists and is essentially unique.
(b) There exist $\rho_1, \rho_2, \rho_3, \rho_4$ with $\rho_1 \leq \rho_2 \leq \rho_3 \leq \rho_4$ such that in introspective equilibrium,

(b1) If $\rho < \rho_1$, then a player with risk parameter $\rho_j$ chooses $H$;
(b2) If $\rho \in (\rho_1, \rho_2)$, then a player chooses $H$ if he belongs to the minority group and chooses the action he expects to be culturally salient otherwise;
(b3) If $\rho \in (\rho_2, \rho_3)$, then every player chooses the action he expects to be culturally salient;
(b4) If $\rho \in (\rho_3, \rho_4)$, then a player chooses $L$ if he belongs to the minority group and chooses the action he expects to be culturally salient otherwise;
(b5) If $\rho > \rho_4$, then every player chooses $L$.

(c) The proportion of players choosing $H$ decreases as the risk of choosing $H$ increases (i.e., $\rho_j$ increases for every $j$).

The proof follows from the proof of Proposition A.2, which, in fact, proved the result for any monotone game with strategic complementarities and identical preferences (which obviously includes semi-convex games with identical preferences.

The following result extends Proposition 3.2 to semi-convex games:

Proposition II.2. [Diversity] For any semi-convex game, players choose action they expect to be culturally salient for larger range of payoffs if the society is homogeneous. That is, in introspective equilibrium, players with risk parameter $\rho_j$ choose the action they expect to be culturally salient if and only if $\rho_j \in (r_L(\beta), r_H(\beta))$, with

$$r_H(\beta = 0) > r_H(\beta = \frac{1}{2}) > r_L(\beta = \frac{1}{2}) > r_L(\beta = 0).$$

with strict inequalities if preference heterogeneity sufficiently small (i.e., players have identical preferences or the variance of $F(\rho_j)$ is sufficiently small). Moreover, the difference between culturally homogeneous and diverse societies is more pronounced when the culture is strong ($q$ close to 1) and the cultural distance between groups is large ($d$ close to 1).

The proof is relegated to Online Appendix VI.
II.1 Application: Industrialization

An example of a semi-convex game that is not linear is the model of industrialization developed by Murphy et al. (1989). This model considers a central question in development economics – which goes back to the seminal work of Rosenstein-Rodan (1943) – under which conditions an economy can become industrialized. There is a continuum of goods \( j \in N \), consumed by a representative consumer. Each good is produced in its own sector, and each sector consists of two types of firms: a competitive fringe of firms that use a constant returns to scale (cottage production) technology and a firm that has access to an increasing returns (mass production) technology. Cottage production yields one one unit of output for each unit of labor input. For each sector \( j \in N \), the firm with access to the increasing returns technology (the “monopolist”) decides whether to industrialize (choose \( H \)) or to abstain from production (choose \( L \)). The increasing returns technology requires a fixed cost of \( F \) units of labor to set up a factory but then yields \( a > 1 \) units of output per unit of labor. The representative consumer has Cobb-Douglas preferences:

\[
V = \begin{cases} 
\exp \left( \int_0^1 \ln(x(j) dj) \right) & \text{if employed in cottage production; } \\
\exp \left( \int_0^1 \ln(x(j) dj) \right) - v & \text{if employed in mass production; }
\end{cases}
\]

where \( x(j) \) is the amount consumed of good \( j \) and \( v > 0 \) represents the disutility from working in a factory. The consumer has \( L \) units of labor which he supplies inelastically. The wage in cottage production is chosen to be the numeraire so that the wage in mass production is \( w = 1 + v \). Aggregate income is the sum of labor income and profits. When a proportion \( m \) of monopolists produce and aggregate income is \( y = y(m) \), the profit for a monopolist is

\[
\pi(m) = y \left( 1 - \frac{1 + v}{a} \right) - F \left( 1 + v \right),
\]

where \( p = 1 \) is the price that the monopolist receives and \( \frac{1 + v}{a} \) is its average variable cost. A monopolist that does not produce receives 0. We assume \( a - 1 > v \) to rule out the trivial case where it is never profitable to industrialize. This is a semi-convex game where the players are the monopolists and

\[
g(m) := \frac{1}{1 - \left( 1 - \frac{1 + v}{a} \right) m}; \quad \text{and} \quad \rho := \frac{L v}{L (v + z) - F (1 + v)}.
\]

III Comparison with correlated equilibrium

We compare the set of all introspective equilibria for a given game (across all societies) to the set of correlated equilibria for the game. Relative to correlated equilibrium, introspective equilibrium has considerable cutting power. A first observation is that for any linear game, the set of introspective equilibria (across all societies) is always a strict subset of the class of correlated equilibria.\(^1\) To make

\(^1\)We thus restrict attention to impulse distributions that are associated with some society (i.e., impulse distributions characterized by \( \beta, q, d \)). Without any restrictions on the class of impulse distributions, any correlated equilibrium is an introspective equilibrium for some impulse distribution. This follows from the revelation principle: fix a correlated equilibrium and take the impulse distribution to be the distribution over action profiles generated by the correlated equilibrium. Then the game has a unique introspective equilibrium
this claim precise, we can identity each society (characterized by \(\beta, q, d\)) with the impulse distribution it generates (Section 2.3). Write \(\Delta\) for the class of impulse distributions that are associated with some society. To be able to compare the set of introspective equilibria (profiles of mappings from impulses to actions) to correlated equilibria (distributions over action profiles), we consider the distributions over action profiles induced by introspective equilibrium. That is, for \(\rho \in [0,1]\) and \(\mu \in \Delta\), let \(\Sigma_\mu(\rho)\) be the set of distributions over action profiles induced by some introspective equilibrium for the society described by \(\mu\) and risk parameter \(\rho\). By Proposition A.2, \(\Sigma_\mu(\rho)\) has at least one element; and for generic values of \(\rho\), it has precisely one element. For \(\rho \in [0,1]\), let \(\Sigma(\rho) = \bigcup_{\mu \in \Delta} \Sigma_\mu(\rho)\). With some abuse of terminology, we refer to \(\Sigma(\rho)\) as the set of introspective equilibria (across all \(\mu \in \Delta\)) for risk parameter \(\rho\). Then, the following claim, which is a corollary of Lemma A.5, shows that introspective equilibrium can always rule out certain behaviors that are consistent with correlated equilibrium:

**Corollary III.1. [The Cutting Power of Introspective Equilibrium (I)]** For any \(\rho \in [0,1]\), the set \(\Sigma(\rho)\) of all introspective equilibria (for some society) is a strict subset of the set \(\mathcal{C}(\rho)\) of all correlated equilibria.

**Proof.** Let \(\rho \in [0,1]\). Then, there is a correlated equilibrium in which all players choose \(H\) as well as a correlated equilibrium in which all players choose \(L\) (this follows because both are pure Nash equilibria). If \(\rho < \frac{1}{2}\), then, by Lemma A.5, for every \(\mu \in \Delta\), there is no introspective equilibrium in \(\Sigma_\mu(\rho)\) such that all players choose \(L\). Likewise, if \(\rho > \frac{1}{2}\), then, by Lemma A.5, for every \(\mu \in \Delta\), there is no introspective equilibrium in \(\Sigma_\mu(\rho)\) such that all players choose \(H\). Finally, if \(\rho = \frac{1}{2}\), then, by Lemma A.5, for every \(\mu \in \Delta\), there is no introspective equilibrium in \(\Sigma_\mu(\rho)\) such that all players choose \(L\), and no introspective equilibrium in \(\Sigma_\mu(\rho)\) such that all players choose \(H\). \(\square\)

A second observation is that in some limiting cases, the set of introspective equilibria (across all societies) collapses to a singleton, as the following corollary of Lemma A.5 demonstrates.

**Corollary III.2. [The Cutting Power of Introspective Equilibrium (II)]** As \(\rho\) goes to 0, 1, or \(\frac{1}{2}\), the set of introspective equilibria (across all societies) converges to a singleton:

(a) As \(\rho \to 0\), the set of introspective equilibria (across all societies) converges to the unique strategy profile where all players choose the high action regardless of their impulse;

(b) As \(\rho \to 1\), the set of introspective equilibria (across all societies) converges to the unique strategy profile where all players choose the low action regardless of their impulse;

(c) As \(\rho \to \frac{1}{2}\), the set of introspective equilibria (across all societies) converges to the unique strategy profile in which all players follow their impulse.

in which all players follow their impulse, and this introspective equilibrium coincides with the original correlated equilibrium. See Myerson (1994) for a version of the revelation principle for complete-information game and a discussion in the context of correlated equilibrium.

\(^2\)Note that also for correlated equilibrium, specifying the risk parameter \(\rho\) is sufficient to pin down the incentive constraints: any two linear games with the same risk parameter have the same set of correlated equilibria.

\(^3\)The limit of a collection of sets is the set-theoretic limit.
Again, the proof follows directly from Lemma A.5. So, in the limit that the risk parameter goes to 0, 1, or $\frac{1}{2}$, the set of introspective equilibria (across all societies) collapses to a singleton, and the limiting introspective equilibrium is independent of sociocultural factors. By contrast, the set of correlated equilibria does not converge to a singleton when the risk parameter goes to 0, 1, or $\frac{1}{2}$.

Instead, it is a continuum (except in trivial cases). To see this, note that for any $\rho \in [0, 1]$, the set of correlated equilibria contains at least the strict Nash equilibria as well as the nonstrict pure Nash equilibrium in which a proportion $\rho$ of players chooses $H$; the claim now follows by noting that, except in knife-edge cases, at least two of these Nash equilibria have different payoff profiles, and the set of correlated equilibrium payoff profiles includes the convex hull of Nash equilibrium payoff profiles.

IV Comparative statics and experimental evidence

This appendix discusses the testable implications of the results in Section 3.2 in more detail and connects them with experimental evidence. We focus on the case of identical preferences ($\rho_j = \rho$ for all $j$) because these games have been the focus of much of the experimental literature, though our predictions extend more generally.

A first prediction is that the proportion of players who choose the high action increases as the risk parameter falls (Proposition 3.3). In particular, introspective equilibrium selects one of the pure Nash equilibria if and only if one of the actions stands out in terms of payoffs (i.e., $\rho$ sufficiently close to 0 or 1). On the other hand, if there is limited asymmetry between the actions in terms of payoffs, both actions are chosen with positive probability and behavior is not consistent with Nash equilibrium. There is considerable experimental evidence for this. For two-player coordination games, Mehta et al. (1994), Straub (1995) and Schmidt et al. (2003), among many others, show that for intermediate values of the risk parameter, behavior is not consistent with Nash equilibrium: players coordinate at a higher rate than in mixed Nash equilibrium, but at a lower rate than in pure Nash equilibrium. Another direct implication is that there can be inefficient lock-in: Players may coordinate on a Pareto-dominated equilibrium. This prediction has received ample experimental support for a range of coordination games (see, e.g., Van Huyck et al., 1990; Cooper et al., 1990, 1992; Straub, 1995).

A second testable implication is that for intermediate values of the risk parameter (i.e., $\rho$ close to $\frac{1}{2}$), behavior is strongly influenced by situational factors (i.e., the cultural salience of actions), and that behavioral consistency improves when strategic uncertainty is reduced. Experimental support for the influence of contextual factors comes from a variety of sources. First, there is extensive evidence that past experience influences strategic behavior even when there are no incentives to build reputation or signal intentions (e.g., Schmidt et al., 2003). To the extent that history shapes impulses, this is consistent with our results. A second type of evidence for this prediction involves the (cultural) saliency of alternatives. Evidence suggests that when the payoff structure of the game provides little guidance (i.e., $\rho$ close to $\frac{1}{2}$) and one action is (culturally) salient, then players have a pre-reflective inclination

\footnote{We are not aware of any experimental studies that study games with extreme values for the risk parameter. This could be a selection effect: if the interest is in testing competing hypotheses, there is no reason to select games for which there is an obvious way to play so that all theories make the same prediction; see Schmidt et al. (2003, p. 285) for comments along these lines.}
to select the salient alternative (e.g., Mehta et al., 1994, p. 659). This means that the coordination rate increases when one of the actions is significantly more salient than others, in line with our model. A third source of evidence that contextual clues influence behavior comes from individual variation in perspective-taking ability. An individual with superior perspective-taking abilities presumably has a highly informative signal about other players’ impulses and will thus be better at coordinating. Curry and Jones Chesters (2012) show that in the pure coordination games of Mehta et al. (1994), subjects with superior perspective-taking ability (as measured by a self-report questionnaire) have a higher probability of coordinating when matched against the population, consistent with our theory.

A third prediction is that it is easier for people to anticipate the actions of members of their own group and that people who belong to the same group are more likely to have the same impulse. This is in line with the experimental evidence of Jackson and Xing (2014), who contrast the behavior of subjects residing in India versus the U.S. in a battle-of-the-sexes game. They find that subjects are better able to predict how subjects of their own group would play. Moreover, the two groups differ in the actions that they take. To the extent that actions are a function of impulses, these findings support our assumption that players from the same group are more likely to have the same impulse and that players find it easier to anticipate the impulses of members of their own group. Consistent with our predictions, Jackson and Xing find that subjects are more successful at coordinating when they are matched with a member of their own group.

Existing equilibrium selection methods cannot account for these findings. For example, in the context of coordination games, payoff dominance selects the same Nash equilibrium independent of the risk parameter, as do team reasoning theories (Sugden, 1993). Risk dominance makes the stark prediction that players coordinate on the efficient action (with probability 1) whenever the risk parameter is less than $\frac{1}{2}$, while they coordinate on the inefficient action whenever the risk parameter is greater than $\frac{1}{2}$. So, risk dominance cannot explain why there can be miscoordination when there is limited asymmetry among the actions, as in the work of Mehta et al. (1994) and others. Since the risk-dominant Nash equilibrium is selected by global games methods (Carlsson and van Damme, 1993), evolutionary models (Young, 1993; Kandori et al., 1993), and quantal response equilibrium (McKelvey and Palfrey, 1995), these methods cannot explain the observed behavior either. This also holds for other concepts. Most notably, Crawford and Haller (1990), in their study of how players use asymmetries in the game to coordinate, derive the stark prediction that players coordinate (with probability 1) whenever there is some asymmetry between actions, no matter how small. By predicting that coordination succeeds only if there is sufficient asymmetry between the actions, our model provides a more nuanced and arguably more realistic view than existing concepts. And while some existing methods, such as risk dominance, the global games selection, and certain learning models, can account for inefficient lock-in, a novel prediction not captured by existing models is that the gap between individual incentives and socially optimal behavior is smaller when there is more strategic uncertainty in the sense that societies that experience more strategic uncertainty can avoid inefficient lock-in for a larger range of payoff parameters.

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5 Mixed Nash equilibrium can also not account for the observed behavior: The coordination rate in Mehta et al.’s (1994) and related experiments lies strictly between that in pure and mixed Nash equilibrium.

6 The noisy introspection model of Goeree and Holt (2004) predicts non-Nash behavior in at least some coordination games. However, it is unclear how predictions vary with payoffs and thus whether the model can reproduce the observed comparative statics.
V  Details for applications

This appendix provides formal statements for the claims related to the applications in Sections 3.2 and shows that all applications satisfies the relevant conditions (Section V.3. All proofs not included here are relegated to Online Appendix VI.

V.1 Excessive conformism

The following is the formal statement of the claim that increasing diversity reduces excessive conformism when the society is homogeneous. Moreover, there is less conformism when the culture is weak and when the cultural distance between groups is large. To state the result, we write \( F(\rho_j; \rho, \tilde{\sigma}) \) for a distribution of risk parameters with mean \( \rho \) and variance \( \tilde{\sigma}^2 \), and we assume that \( F(\rho_j; \rho, \tilde{\sigma}) \) is unimodal and symmetric.

Corollary V.1. [Excessive Conformism] If there is limited preference heterogeneity, then the set of risk parameters for which there is excessive conformism decreases with diversity \( \beta \) for \( \beta \) sufficiently small provided \( y \). Moreover, the set decreases with cultural distance \( d \) and increases with culture strength \( q \). That is, given \( \beta, \beta' \) with \( \beta' > \beta \), if \( \beta' \) is sufficiently small, then there is \( \tilde{\sigma}^* \) such that for \( \tilde{\sigma} \leq \tilde{\sigma}^* \), for any \( \rho \) and \( F(\rho_j; \rho, \tilde{\sigma}) \), if there is excessive conformism when diversity is \( \beta' \), then there is excessive conformism when diversity is \( \beta \); similar statements hold when we replace \( \beta, \beta' \) by \( q, q' \) and \( d, d' \) (with \( q < q' \) and \( d > d' \)) respectively.

V.2 Corruption

The following result formalizes the claim that staff rotation schemes and other measures to combat corruption are complements. To state the result, assume that group \( A \) consists of “oldtimers,” i.e., officials who have been in post for a long period in time, while group \( B \) (with share \( \beta \)) are the “newcomers.”

Corollary V.2. [Anti-Corruption Measures Are Complements] Increasing officials’ wage and improving the probability of detection is more effective at eliminating corruption when a staff rotation scheme is in place; and, conversely, introducing a staff rotation scheme is more effective when other anti-corruption measures are in place. That is, suppose \( w, p, c, p \) and \( \beta \) are such that there is corruption both in the absence of a staff rotation scheme (no newcomers) and with the staff rotation scheme in place (with share \( \beta \) of newcomers). Then there exist \( w' \geq w \) and \( p' \geq p \) such that if the wage and detection probability increase to \( w' \) and \( p' \), respectively, then corruption is eliminated under the staff rotation scheme but not without it.

The proof follows from Proposition 3.3 and is therefore omitted.

V.3 Welfare

We next document that all applications in Section 3 with identical preferences satisfy the conditions for Proposition 3.4. That is, we show that for each application, the social welfare function \( W(m; \rho) \) is
quadratic in $m$ with its minimum $\underline{m}$ increasing in $\rho$, or, equivalently, that $W(1; \rho) - W(0; \rho)$ decreases with $m$.

We start with the example in Section 3.1 and the related model of excessive conformism. These are linear games, with the risk parameter for player $j$ equal to $\rho_j = \frac{1}{2} + \frac{\lambda}{2(1-\lambda)}(1 - 2\tau_j)$ (where $\lambda$ and $\tau$ are set equal to $\frac{1}{2}$ and $\tau$ in Section 3.1, respectively). If $\rho_j = \rho$ for all $j$ (as in Section 3.1), then the social welfare function is quadratic with minimum $\underline{m} = \rho$. Similar comments apply for the model of organizational persistence in Section 3.3.3.

The model of corruption is a linear game with identical preferences with risk parameter $\rho = \frac{1}{2} - \frac{pw}{b - c}$. Social welfare is convex and quadratic in $m$, and $W(1; \rho) - W(0; \rho) = -pw\rho$. So, $W(1; \rho) - W(0; \rho)$ decreases with $\rho$ if $p$ and $w$ are fixed (but $b$ and $c$ are varied).

The infinitely repeated game in Section 3.3.1 is a linear game with identical preferences. To show that it satisfies the relevant conditions, we show that this is true for general $(2 \times 2)$ coordination games:

$$
\begin{array}{cc}
H & L \\
H & u_{HH} & u_{HL} \\
L & u_{LH} & u_{LL}
\end{array}
$$

with $u_{HH} > u_{LH}$, $u_{LL} > u_{HL}$, and $u_{HH} \geq u_{LL}$. The infinitely repeated game is then a special case (with, e.g., $u_{cc} = u_{HH}$). Coordination games satisfy the conditions in Proposition 3.4 if $2u_{LL} > u_{LH} + u_{HL}$ (which holds if cooperation is efficient, i.e., $2u_{cc} > u_{cd} + u_{dc}$) and we are considering, e.g., increasing $u_{LL}$ to $u_{HH}$ keeping the other payoff parameters fixed.

The model of cultures of dependency in Section 3.3.2 satisfies the conditions in Proposition 3.4 when all players receive the same wage: Welfare is convex and quadratic in $m$, and $W(1; \rho) - W(0; \rho) = -c\rho$ (where $c > 0$).

VI Omitted proofs

VI.1 Proof of Proposition A.2 (cntd)

We prove existence of introspective equilibrium for the case of heterogeneous preferences where the distribution $F(\rho_j)$ has mean $\mu \geq \frac{1}{2}$. As for the case $\mu \leq \frac{1}{2}$, we prove the result under slightly weaker assumptions than in the main text: rather than assuming that $f(\rho_j)$ is unimodal and symmetric, we require that the density $f(\rho_j)$ satisfies

$$
\begin{align*}
&f(\frac{1}{2} + x) \geq f(\frac{1}{2} + y) & \forall x, y \text{ s.t. } y \geq x \geq 0; \\
&f(\frac{1}{2} - x) \geq f(\frac{1}{2} + x) & \forall x \geq 0.
\end{align*}
$$

(VI.1) (VI.2)
Again, these conditions are satisfied when \( f(\rho_j) \) is unimodal and symmetric (with mean \( \mu \geq \frac{1}{2} \)) but they are strictly weaker. As before, we can rewrite the expressions for \( \rho_{HA}^k \) and \( \rho_{HB}^k \) as

\[
\rho_{HA}^k = \beta(Q_m - \bar{Q}_m)F(\rho_{HA}^{k-1}) + \beta \bar{Q}_m[F(\rho_{HA}^{k-1}) + F(\rho_{LA}^{k-1})] + \\
\beta(Q_o - \bar{Q}_o)F(\rho_{HA}^{k-1}) + \beta \bar{Q}_o[F(\rho_{HA}^{k-1}) + F(\rho_{LB}^{k-1})];
\]
\[
\rho_{HB}^k = \beta(Q_o - \bar{Q}_o)F(\rho_{HB}^{k-1}) + \beta \bar{Q}_o[F(\rho_{HA}^{k-1}) + F(\rho_{LB}^{k-1})] + \\
\beta(Q_m - \bar{Q}_m)F(\rho_{HB}^{k-1}) + \beta \bar{Q}_m[F(\rho_{HB}^{k-1}) + F(\rho_{LB}^{k-1})];
\]

respectively. So, by a similar argument as before, it suffices to prove that \( \{\rho_{HA}^k\}, \{\rho_{HB}^k\}, \) and \( \{\rho^k\} \) converge. This follows from the following analogues of Lemmas A.3–A.4:

**Lemma VI.1.** Suppose \( f(\rho_j) \) has mean \( \mu \geq \frac{1}{2} \) and satisfies (VI.1)–(VI.2), and fix a group \( G \in \{A,B\} \) and \( k > 0 \). If \( \rho^k \leq \rho^{k-1} \leq \frac{1}{2} \) and \( \rho_{LG}^k \leq \rho_{LG}^{k-1} \), then \( F(\rho_{HG}^k) + F(\rho_{LG}^k) \leq F(\rho_{HG}^{k-1}) + F(\rho_{LG}^{k-1}) \).

**Proof.** For concreteness, take \( G = A \). If \( \rho_{LA}^k \geq \rho_{LA}^{k-1} \), then the result follows immediately from the fact that \( F(\rho_j) \) is increasing. So suppose that \( \rho_{LA}^k < \rho_{LA}^{k-1} \). Define \( \Delta^k := \rho_{LA}^k - \rho_{LA}^{k-1} \). By a similar argument as before,

\[
0 < \Delta^k \leq \rho_{HA}^{k-1} - \rho_{HA}^k
\]

and

\[
[F(\rho_{HA}^k) + F(\rho_{LA}^k)] - [F(\rho_{HA}^{k-1}) + F(\rho_{LA}^{k-1})] \\
\geq \int_0^{\Delta^k} [f(\rho^k + (\rho_{HA}^k - \rho_{LA}^k) + u) - f(\rho^{k-1} - (\rho_{HA}^k - \rho_{LA}^k) + u)] du.
\]

The result then follows by noting that, under (VI.1)–(VI.2) (and \( \mu \geq \frac{1}{2} \)), for any \( \bar{\rho} \leq \frac{1}{2} \) and \( x \geq 0 \), \( f(\bar{\rho} + x) \geq f(\bar{\rho} - x) \).

To prove the result for \( k = 1 \), it suffices to show that for \( \bar{\rho} \leq \frac{1}{2} \) and \( x \geq 0 \), \( F(\bar{\rho} + x) + F(\bar{\rho} - x) \leq 1 \). But this follows from a similar argument as before, using that \( f(\bar{\rho} - y) \leq f(\bar{\rho} + y) \) for \( y \geq 0 \) and \( \bar{\rho} \leq \frac{1}{2} \). □

**Lemma VI.2.** Suppose \( f(\rho_j) \) has mean \( \mu \geq \frac{1}{2} \) and satisfies (VI.1)–(VI.2). Then, for all \( k > 0 \), \( \rho^k \leq \rho^{k-1} \leq \frac{1}{2} \), \( \rho_{HA}^k \leq \rho_{HA}^{k-1} \) and \( \rho_{HB}^k \leq \rho_{HB}^{k-1} \).

The proof is analogous to that of Lemma A.4 and thus omitted. It now follows immediately that the sequences \( \{\rho_{HA}^k\}, \{\rho_{HB}^k\}, \) and \( \{\rho^k\} \) converge: As before, by Lemma VI.2, each sequence is bounded and monotone. Hence, there exist \( \bar{\rho} \in (0, \frac{1}{2}) \), \( \rho_{HA} \in [\bar{\rho}, 1) \), and \( \rho_{HB} \in [\bar{\rho}, 1) \) such that \( \rho^k \downarrow \bar{\rho} \), \( \rho_{HA} \downarrow \rho_{HA} \), and \( \rho_{HB} \downarrow \rho_{HB} \). □

**VI.2 Proof of Proposition B.1**

The proof for threshold games with identical preference follows from the proof for games with identical preferences (Appendix A.2.1). So suppose that there is preference heterogeneity, i.e., players’ risk parameters are distributed according to a continuous distribution \( F(\rho_j) \).

We start with part (a). We first prove existence. We write \( I_E \) for the indicator function for the event \( E \). That is, \( I_E = 1 \) if \( E \) obtains, and \( 0 \) otherwise. Also, we define \( F(\infty) := \lim_{x \to \infty} F(x) \) and \( F(-\infty) := \lim_{x \to -\infty} F(x) \), and we write \( z := \frac{1}{2}(1 + \eta) \) (and thus \( z := 1 - z = \frac{1}{2}(1 - \eta) \)).

Again, the level-0 strategy is a switching strategy: At level 0, player of type \( (I_j, G_j, \rho_j) \) chooses \( H \) if \( \rho_j < \rho_{I_jG_j}^0 \) and chooses \( L \) if \( \rho_j > \rho_{I_jG_j}^0 \) (a type with \( \rho_j = \rho_{I_jG_j}^0 \) can choose either action). Then,
we can summarize the level-0 by \( (\rho^0_{HA}, \rho^0_{HB}, \rho^0_{LB}, \rho^0_{LA}) := (\infty, \infty, -\infty, -\infty) \). For \( k > 0 \), suppose that there exist cutoffs \( (\rho_{HA}^{k-1}, \rho_{HB}^{k-1}, \rho_{LB}^{k-1}, \rho_{LA}^{k-1}) \) with \( \rho^0_{HA} \geq \rho^0_{HB} \geq \rho^0_{LB} \geq \rho^0_{LA} \), such that, at level \( k - 1 \), every type \( (I, G, \rho_j) \) chooses \( H \) if \( \rho_j < \rho^0_{IG} \) and chooses \( L \) if \( \rho_j > \rho^0_{IG} \). (As before, if \( \rho_j = \rho^0_{IG} \), the type may choose either action.) Then, at level \( k \), \( H \) is the unique best response for type \( (I, G, \rho_j) \) if and only if

\[
\rho_j < \mathbb{P}(m \geq T | I, G) =: \rho^k_{IG},
\]

where \( \mathbb{P}(m \geq T | I, G) \) is the conditional probability that a player from group \( G \) with impulse \( I \) assigns to the event that the proportion \( m \) of players who choose \( H \) is at least as high as the threshold (given the level-\( k - 1 \) strategies given by the cutoffs \( (\rho_{HA}^{k-1}, \rho_{HB}^{k-1}, \rho_{LB}^{k-1}, \rho_{LA}^{k-1}) \)). Likewise, \( L \) is the unique best response for type \( (I, G, \rho_j) \) if and only if \( \rho_j > \rho^k_{IG} \). Consequently, at level \( k \), players follow a switching strategy with cutoffs \( (\rho_{HA}^k, \rho_{HB}^k, \rho_{LB}^k, \rho_{LA}^k) \), where \( \rho^k_{IG} = \mathbb{P}(m \geq T | I, G) \).

We can derive explicit expressions for the cutoffs. It will be helpful to introduce the notation \( \pi_{IG} = (\pi^I_{IG}, \pi^H_{IG}, \pi^L_{IG}, \pi^T_{IG}) \) for the conditional beliefs of players over states, with \( \pi^0_{IG} \) is the conditional probability that a player from group \( G \) with impulse \( I \) assigns to state \( (\theta_A, \theta_B) = (\theta, \theta') \), i.e.,

\[
\pi_{HA} := (qz, q\tilde{z}, \tilde{q}z, \tilde{q}z); \quad \pi_{HB} := (\tilde{q}z, q\tilde{z}, \tilde{q}z, qz); \quad \pi_{LA} := (\tilde{q}z, q\tilde{z}, q\tilde{z}, qz);
\]

Then, if we denote by \( m^{k-1}_{\theta_0} \) the proportion of players who choose \( H \) if \( (\theta_A, \theta_B) = (\theta, \theta') \), we have

\[
\begin{align*}
\rho_{HA}^k &= q \mathbb{I}_{[m_{HA}^{k-1} \geq T]} + q \mathbb{I}_{[m_{HL}^{k-1} \geq T]} + q \mathbb{I}_{[m_{LA}^{k-1} \geq T]} + q \mathbb{I}_{[m_{HH}^{k-1} \geq T]}; \\
\rho_{HB}^k &= q \mathbb{I}_{[m_{HA}^{k-1} \geq T]} + q \mathbb{I}_{[m_{HL}^{k-1} \geq T]} + q \mathbb{I}_{[m_{LA}^{k-1} \geq T]} + q \mathbb{I}_{[m_{HH}^{k-1} \geq T]}; \\
\rho_{LB}^k &= \tilde{q} \mathbb{I}_{[m_{HA}^{k-1} \geq T]} + q \mathbb{I}_{[m_{HL}^{k-1} \geq T]} + q \mathbb{I}_{[m_{LA}^{k-1} \geq T]} + q \mathbb{I}_{[m_{HH}^{k-1} \geq T]}; \\
\rho_{LA}^k &= \tilde{q} \mathbb{I}_{[m_{HA}^{k-1} \geq T]} + q \mathbb{I}_{[m_{HL}^{k-1} \geq T]} + q \mathbb{I}_{[m_{LA}^{k-1} \geq T]} + q \mathbb{I}_{[m_{HH}^{k-1} \geq T]};
\end{align*}
\]

where

\[
\begin{align*}
m_{HH}^{k-1} &= \tilde{\beta} q F(\rho_{HA}^{k-1}) + \beta q F(\rho_{HB}^{k-1}) + \beta \tilde{q} F(\rho_{LB}^{k-1}) + \tilde{\beta} q F(\rho_{LA}^{k-1}); \\
m_{HL}^{k-1} &= \tilde{\beta} q F(\rho_{HA}^{k-1}) + \beta q F(\rho_{HB}^{k-1}) + \beta \tilde{q} F(\rho_{LB}^{k-1}) + \tilde{\beta} q F(\rho_{LA}^{k-1}); \\
m_{HK}^{k-1} &= \tilde{\beta} q F(\rho_{HA}^{k-1}) + \beta q F(\rho_{HB}^{k-1}) + \beta \tilde{q} F(\rho_{LB}^{k-1}) + \tilde{\beta} q F(\rho_{LA}^{k-1}); \\
m_{LL}^{k-1} &= \tilde{\beta} q F(\rho_{HA}^{k-1}) + \beta q F(\rho_{HB}^{k-1}) + \beta \tilde{q} F(\rho_{LB}^{k-1}) + \tilde{\beta} q F(\rho_{LA}^{k-1}).
\end{align*}
\]

By the induction hypothesis, \( m_{HA}^{k-1} \geq m_{HB}^{k-1} \geq m_{HL}^{k-1} \geq m_{LA}^{k-1} \), and (using that \( \tilde{\beta} \geq \beta \) and \( z > \tilde{z} \)),

\[
\rho_{HA}^k \geq \rho_{HB}^k \geq \rho_{LB}^k \geq \rho_{LA}^k.
\]

Establishing existence – i.e., showing that the cutoffs \( (\rho_{HA}^k, \rho_{HB}^k, \rho_{LB}^k, \rho_{LA}^k) \) converges to some vector \( (\rho_{HA}, \rho_{HB}, \rho_{LB}, \rho_{LA}) \) as \( k \to \infty \) – is facilitated by the fact that the indicator function can take on only two values, 0 and 1: At each level \( k \geq 1 \), \( (\rho_{HA}^k, \rho_{HB}^k, \rho_{LB}^k, \rho_{LA}^k) \in \mathcal{R} := \{R_1, R_2, R_3, R_4, R_5\} \), where the five vectors

\[
\begin{align*}
R_1 &:= (0, 0, 0, 0); \\
R_2 &:= (qz, q\tilde{z}, \tilde{q}z, \tilde{q}z); \\
R_3 &:= (q, qz + \tilde{q}z, \tilde{q}z + q\tilde{z}, \tilde{q}z); \\
R_4 &:= (1 - qz, 1 - q\tilde{z}, 1 - qz, 1 - q\tilde{z}); \\
R_5 &:= (1, 1, 1, 1);
\end{align*}
\]

in \( \mathcal{R} \) correspond to the configurations for \( (\mathbb{I}_{[m_{HA} \geq T]}), \mathbb{I}_{[m_{HL} \geq T]}, \mathbb{I}_{[m_{LA} \geq T]}, \mathbb{I}_{[m_{HH} \geq T]} \) in \( \{0, 1\}^4 \) that are consistent with \( m_{HA} \geq m_{HL} \geq m_{LL} \geq m_{LA} \). Importantly, the vectors in \( \mathcal{R} \) can be ordered: \( R_1 < R_2 < R_3 < R_4 < R_5 \). It is now immediate that the process converges: write
\( \rho^k := (\rho_{HA}^k, \rho_{HB}^k, \rho_{LB}^k, \rho_{LA}^k) \). Then, for \( k \geq 1 \), either (i) \( \rho^{k+1} \geq \rho^k \); or (ii) \( \rho^{k+1} \leq \rho^k \). If \( \rho^{k+1} \geq \rho^k \), then, by strategic complementarities, \( \rho^{k+1} \geq \rho^l \) for all \( l \geq 1 \); and if \( \rho^{k+1} \leq \rho^k \), then \( \rho^{k+1} \leq \rho^l \) for all \( l \geq 1 \). Moreover, for \( k \geq 1 \), we have \( \rho^k \in [0,1]^4 \). So, we have a monotone sequence in a bounded space, which must converge. This proves existence. The proof that the introspective equilibrium is essentially unique again follows from the fact that the best response for a type with risk parameter \( \rho \) is unique for a set of risk parameters with measure 1 (under \( F(\rho) \)).

The proof of (b) again follows by setting \( \rho^1 := \rho_{LA}, \rho^2 := \rho_{LB}, \rho^3 := \rho_{HB}, \) and \( \rho^4 := \rho_{HA} \) and noting that \( \rho_{HA} \geq \rho_{HB} \geq \rho_{LB} \geq \rho_{LA} \).

**VI.3 Proof of Lemma B.2**

Recall the notation \( (\pi_{1G}^{HH}, \pi_{1G}^{HL}, \pi_{1G}^{LH}, \pi_{1G}^{LL}) \) for the conditional beliefs of players over states, where \( \pi_{1G}^{\theta_0'} \) is the conditional probability that a player from group \( G \) with impulse \( I \) assigns to state \( (\theta_A, \theta_B) = (\theta, \theta_0') \).

If \( T \leq m_{LL}^0 \), then, for any \( \rho < 1 \), the unique best response at level 1 is to attack: at level 0, the proportion of players with an impulse to attack is at least \( m_{LL}^0 \) in any state \( (\theta_A, \theta_B) \); so, any attack will be successful. If \( \rho > 1 \), then players have a strictly dominant strategy not to attack. Clearly, at level 1, all players are playing best responses to others’ (level-1) strategies, so this describes the introspective equilibrium.

Next suppose that \( T \in (m_{LL}^0, m_{LL}^0) \). Then, at level 0, an attack is successful if and only if \( (\theta_A, \theta_B) \neq (L, L) \) (i.e., if attacking is culturally salient for at least one group). If \( \rho < 1 - \pi_{LA}^{LL} \), then the unique best response at level 1 is to attack; if \( \rho \in (1 - \pi_{LA}^{LL}, 1 - \pi_{HA}^{LL}) \), then the unique best response at level 1 for players is to choose the action they expect to be culturally salient; and if \( \rho > 1 - \pi_{LA}^{LL} \), then the unique best response at level 1 is to not attack. Again, at level 1, all players are playing best responses to others’ (level-1) strategies, so this describes the introspective equilibrium.

We next consider \( T \in (m_{LL}^0, m_{LL}^0) \). Then, at level 0, an attack is successful if and only if \( \theta_A = H \) (i.e., if attacking is culturally salient for group \( A \)). If \( \rho < 1 - \pi_{LA}^{LL} - \pi_{HA}^{LL} \), then the unique best response at level 1 is to attack; if \( \rho \in (1 - \pi_{LA}^{LL} - \pi_{LB}^{LL} - \pi_{HA}^{LL} - \pi_{HB}^{LL}) \), then the unique best response at level 1 for players is to choose the action they expect to be culturally salient; and if \( \rho > 1 - \pi_{LA}^{LL} - \pi_{HA}^{LL} \), then the unique best response at level 1 is to not attack. In each of these cases, all players are playing best responses to others’ (level-1) strategies, so the introspective equilibrium coincides with the level-1 strategies. It remains to consider the cases \( \rho \in (1 - \pi_{LA}^{LL} - \pi_{LB}^{LL} - \pi_{HA}^{LL} - \pi_{HB}^{LL}) \) and \( \rho \in (1 - \pi_{LA}^{LL} - \pi_{LB}^{LL} - \pi_{HA}^{LL} - \pi_{HB}^{LL}) \). In the first case (\( \rho \in (1 - \pi_{LA}^{LL} - \pi_{LB}^{LL} - \pi_{HA}^{LL} - \pi_{HB}^{LL}) \)), the unique best response for majority players is to choose the action they expect to be culturally salient, while the unique best response for minority players is to attack. This need not be an introspective equilibrium, so we need to consider the level-2 strategies. At level 2, all players play a best response against their belief except majority players with an impulse to choose \( L \), i.e., type \( (L, A) \). At level 2, \( H \) is the unique best response for \( (L, A) \) if and only if \( q_{[1-\beta q \geq T]} + q_{[1-\beta q \geq T]} > \rho \), which holds if and only if \( 1 - \beta q \geq T \). Hence, at level 2, either all players attack (if \( T \) is sufficiently small), or minority players attack while majority players attack if and only if they expect attacking to be culturally salient; in either case, all types play a best response against others’ level-2 strategies, so we have an introspective equilibrium. The proof for the second case \( \rho \in (1 - \pi_{HB}^{LL} - \pi_{HB}^{LL} - \pi_{LA}^{LL} - \pi_{LA}^{LL}) \) is similar and thus omitted.

11
Next suppose that $T \in (m_{HL}^0, m_{HH}^0]$. Then, at level 0, an attack is successful if and only if $(\theta_A, \theta_B) = (H, H)$ (i.e., if attacking is culturally salient for both groups). If $\rho < \pi_{LA}^{HH}$, then the unique best response at level 1 is to attack; if $\rho \in (\pi_{LA}^{LL}, \pi_{LA}^{HH})$, then the unique best response at level 1 for players is to choose the action they expect to be culturally salient; and if $\rho > \pi_{LA}^{HH}$, then the unique best response at level 1 is to not attack. Again, at level 1, all players are playing best responses to others’ (level-1) strategies, so this describes the introspective equilibrium.

Finally, suppose that $T > m_{HH}^0$. Then, for any $\rho > 0$, the unique best response at level 1 is to not attack: at level 0, the proportion of players with an impulse to attack is at most $m_{HH}^0$ for any state $(\theta_A, \theta_B)$; so, no attack can be successful. If $\rho < 0$, then players have a strictly dominant strategy to attack. Clearly, at level 1, all players are playing best responses to others’ (level-1) strategies, so this describes the introspective equilibrium. □

VI.4 Proof of Proposition B.3

For threshold games with identical preferences with $\beta \in \{0, \frac{1}{2}\}$, we have $r_I(\beta) = \mathbb{P}_\beta(m \geq T \mid I)$, where $\mathbb{P}_\beta(m \geq T \mid I)$ is the conditional probability that a player with impulse $I$ assigns to the event that the proportion of players who choose $H$ at level 0 is at least $T$, given that his impulse is $I$. By Lemma B.2,

$$
\mathbb{P}_{\beta=0}(m \geq T \mid H) = \begin{cases} 
1 & \text{if } T \leq 1 - q; \\
q & \text{if } T \in (1 - q, q]; \\
0 & \text{otherwise};
\end{cases}
$$

$$
\mathbb{P}_{\beta=0}(m \geq T \mid L) = \begin{cases} 
1 & \text{if } T \leq 1 - q; \\
1 - q & \text{if } T \in (1 - q, q]; \\
0 & \text{otherwise};
\end{cases}
$$

$$
\mathbb{P}_{\beta=\frac{1}{2}}(m \geq T \mid H) = \begin{cases} 
1 & \text{if } T \leq 1 - q; \\
1 - \frac{1-q}{2}(1 + \eta) & \text{if } T \in (1 - q, \frac{1}{2}]; \\
\frac{q}{2}(1 + \eta) & \text{if } T \in (\frac{1}{2}, q]; \\
0 & \text{otherwise};
\end{cases}
$$

$$
\mathbb{P}_{\beta=\frac{1}{2}}(m \geq T \mid L) = \begin{cases} 
1 & \text{if } T \leq 1 - q; \\
1 - \frac{q}{2}(1 + \eta) & \text{if } T \in (1 - q, \frac{1}{2}]; \\
\frac{1-2q}{2}(1 + \eta) & \text{if } T \in (\frac{1}{2}, q]; \\
0 & \text{otherwise}.
\end{cases}
$$

So, for $T \in (1 - q, q]$, $\Delta(\beta = 0) := r_H(\beta = 0) - r_L(\beta = 0) = 2q - 1 > \frac{2-1}{2}(1 + \eta) = r_H(\beta = \frac{1}{2}) - r_L(\beta = \frac{1}{2}) =: \Delta(\beta = \frac{1}{2})$, while for $T \not\in (1 - q, q]$, $\Delta(\beta = 0) = \Delta(\beta = \frac{1}{2}) = 0$. The proof of the second part of Proposition B.3 follows from the observation that $\Delta(\beta = 0) - \Delta(\beta = \frac{1}{2})$ is increasing in $q$ and $d = 1 - \eta$. □

VI.5 Proof of Proposition II.2

We prove the result for semi-convex games with identical preferences. By the proof of Proposition A.2 (Appendix A.2.1), if $\beta = 0$ or $\beta = \frac{1}{2}$, then, in introspective equilibrium, a player with impulse
I from group $G$ chooses $H$ whenever $E_\beta[g(m) \mid I] > \rho$ and chooses $L$ if $E_\beta[g(m) \mid I] < \rho$, where $E_\beta[g(m) \mid I]$ is the conditional expectation of $g(m)$ at level 1 for a player with impulse $I$. Notice that this conditional expectation does not depend on the player’s group (i.e., $E_\beta[g(m) \mid I,G] = E_\beta[g(m) \mid I]$ for each $G$): either both groups have the same conditional beliefs (if $\beta = \frac{1}{2}$, by symmetry) or there is only one group (if $\beta = 0$). So, $r_I(\beta) = E_\beta[g(m) \mid I]$. We claim that $r_L(\beta = 0) < r_L(\beta = \frac{1}{2})$ and $r_H(\beta = 0) > r_H(\beta = \frac{1}{2})$. This is equivalent to

$$g(\frac{1}{2}) > gg(1-q) + (1-q)g(q);$$

$$g(\frac{1}{2}) < gg(q) + (1-q)g(1-q).$$

But this follows directly from the definition of semi-convex games (using Jensen’s inequality and $q^2 + (1-q)^2 > \frac{1}{2}$). We have $\Delta(\beta = 0) := r_H(\beta = 0) - r_L(\beta = 0) = (2q - 1) (g(q) - g(1-q)) + \frac{2q-1}{2} > 0$ and the first part of Proposition II.2 follows. The second part follows by noting that $\Delta(\beta = 0) - \Delta(\beta = \frac{1}{2}) = \frac{2q-1}{2} (1-\eta) (g(q) - g(1-q))$ is increasing in $d$ and $q$.

**VI.6 Proof of Corollary V.1**

For games with identical preferences, the result follows from Lemma A.5. Collective misrepresentation occurs when some players choose the option they expect to be culturally salient (parts (b)–(e) in Lemma A.5). For $\beta < \beta^*$, the set of payoff parameters for which players choose the action they expect to be culturally salient increases with $\beta$, where $\beta^* = \frac{Q_{in} - Q_{out}}{2Q_{in} - Q_{out}}$ solves $1 - \tilde{\beta} Q_{in} = \tilde{\beta} Q_{out} + \beta Q_{in}$; or, equivalently, $\tilde{\beta} Q_{in} = \tilde{\beta} Q_{out} + \beta Q_{in}$. That is, $\beta^*$ is the level of diversity at which the range of risk parameters for which players choose the action they expect to be culturally salient is minimized (Figure 2). The comparative statics on $q$ and $d$ follow by noting that $Q_{in}$ and $Q_{out}$ are increasing in $q$ while $Q_{out}$ decreases with $d$. Hence, as $q$ increases or $d$ decreases, the boundaries $1 - \tilde{\beta} Q_{in}$ and $\tilde{\beta} Q_{out} + \beta Q_{in}$ shift down (equivalently, the boundaries $\tilde{\beta} Q_{in}$ and $\tilde{\beta} Q_{out} + \beta Q_{in}$ shift up).

By Lemma A.6, these results extend to games with preference heterogeneity provided the variation in preferences is sufficiently small.

**References**


