How (Not) to Eat a Hot Potato*

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Abstract

We consider a dynamic matching problem where players are repeatedly assigned tasks and can choose whether to accept or reject them. Players prefer to avoid certain tasks ("hot potatoes") while other tasks give a positive payoff ("sweet potatoes"). There are frictions in the matching process in that players may not be matched to desirable tasks even if one is available. Both under the optimal mechanism and in decentralized settings, players may accept hot potatoes if this reduces frictions in the matching process. We quantify the welfare loss due to matching frictions and show that, unlike losses due to more conventional frictions, it does not vanish when the cost of handling hot potatoes grows large.

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1 Introduction

With tasks and employers stressed out about traffic jams and unreliable transit service, this political hot potato is back on Beacon Hill. – The Boston Globe, 7 Nov. 2019

Many matching processes involve frictions, with participants not being matched with others even if such matches would generate value. For example, this is a driving force behind the rise of matching platforms that facilitate the search for partners. The effects of frictions can be aggravated when matching is dynamic if past decisions can increase or reduce frictions in future periods. But despite this importance of frictions to matching, the extant literature on dynamic matching largely abstracts away from matching frictions by assuming that feasible matches that create value will be realized.[1] This paper therefore studies the economic implications of frictions on matching outcomes.

We study the impact of matching frictions in the context of handling hot potatoes. A hot potato is an issue, problem, or person that nobody wants to deal with and is therefore often tossed around. An example are political hot potatoes such as the increases in gas taxes referred to in the quote above (Chesto, 2019). Other examples of political hot potatoes include pension reform in European countries (Minder, 2011), energy subsidies in developing countries (The Economist, 2015), the refugee crisis in Europe (Rohac, 2016), property tax reform (Geringer-Sameth, 2019), exemptions from military service for yeshiva students in Israel (The Economist, 2019b), an export ban on Huawei (Binnie, 2015), regulating political advertising (The Economist, 2019a), and union disputes (The Economist, 2012). Beyond the political realm, examples of games of hot potato include employees responding to email queries that require effort to resolve by firing off short (and not particularly helpful) messages in an effort to put the ball in a co-worker's court, avoiding liability for harassment claims (Wang, 2019), trading subprime mortgages (Wall Street Journal, 2007), and avoiding admitting patients by referring them to others (Newman, 2006).

These examples share two important features. First, there are indirect effects: If an agent kicks the can down the road and does not resolve the problem, it may resurface in the future. For example, if a politician does not tackle a thorny issue (say, so as not to jeopardize his chances of being re-elected), it may come up in the future, as in the quote above. Likewise, an employee who successfully avoids dealing with a problem today by passing it onto a coworker may find the ball back in his court tomorrow.

The second key feature is that there are frictions in the matching process. For example, while politicians can try to set the agenda, they are often unsuccessful because the issues

[1] Whether a match is feasible may of course depend on strategic considerations.

they have to deal with are often driven by external shocks outside their control. So, even if there are issues they would like to take on (say, to increase their popularity), they may be unable to do so. Importantly, these frictions interact with the indirect effects: A decision not to handle a particular hot potato can have important "downstream" consequences: A politician who kicks the can down the road by not resolving a difficult issue today may be forced to face it again when it resurfaces in a future period.

To capture these key features, we keep the baseline model deliberately simple and abstract from features such as direct externalities, private information, or assortative matching. We consider two long-lived players (say, politicians or coworkers). In each period $t = 0, 1, \ldots$, a new task arrives and is randomly assigned to one of the players. The player who is assigned the task can decide to handle ("accept") it or to reject it. There are two types of tasks, referred to as hot potatoes and sweet potatoes. Rejecting a task gives a stage payoff $\underline{u}$, normalized to 0 (independent of the type of task). Accepting a sweet potato gives the player a stage payoff $u_s > 0$ while accepting a hot potato gives the player a stage payoff of $u_h < 0$. Players aim to maximize their discounted sum of payoffs (with common discount factor $\beta \in (0, 1)$). The type of tasks that are available, assignments of tasks to players, and payoff parameters are common knowledge, and players' decisions are publicly observed.

Our first main result characterizes the outcome under decentralized matching where players accept or reject tasks to maximize their discounted sum of payoffs (given the other player's strategy). That is, players play according to an equilibrium of the game. As is standard, we focus on Markov perfect equilibria. A key strategic variable is the relative cost $\rho := |u_h/u_s|$ of handling hot potatoes. If the relative cost of handling hot potatoes is high, then, under decentralized matching, no player accepts the potatoes. If the relative cost of hot potatoes is low, then players accept new hot potatoes (i.e., hot potatoes that have just become available) but not old ones (i.e., hot potatoes that arrived in a previous period). Crucially, there is also an intermediate range of costs where there are multiple possible outcomes under decentralized matching: There is a Markov perfect equilibrium in which both players accept new hot potatoes as well as a Markov perfect equilibrium in which both players reject hot potatoes.[2]

So, under decentralized matching, players may decide to accept hot potatoes even though they dislike them. The intuition is that hot potatoes, when not dealt with, may resurface the next period. This aggravates frictions in the matching process: it reduces the chance that players are matched with a desirable task in the next period. This gives players an incentive to accept the hot potatoes even if that imposes a direct cost on them.

[2] In addition, there is also a mixed Markov perfect equilibrium where new hot potatoes are accepted with a probability strictly between 0 and 1.

However, taken by itself, this may not be enough: For intermediate levels of cost, the prospect of potentially facing a more attractive set of tasks tomorrow is not sufficient for a player to accept the hot potato today if the other does not accept any hot potatoes. However, when the other player does accept hot potatoes, then it is optimal for the player to do the same. So, there are strategic complementarities. Strategic complementarities arise through indirect effects on the matching process: When the other player also accepts hot potatoes, then accepting a hot potato today improves the set of available tasks not only tomorrow but also in future periods. This gives rise to a coordination problem, and therefore to multiplicity.

We next consider the optimal matching protocol when the designer can determine players' acceptance decisions but cannot eliminate the matching frictions directly. Under the optimal matching protocol, players accept hot potatoes for a larger range of cost parameters. In particular, when there are strategic complementarities, the optimal matching protocol selects the outcome where all players accept (new) hot potatoes. In addition, there is a range of payoff parameters where players always reject new hot potatoes under decentralized matching but accept them under the optimal matching protocol. Intuitively, there are externalities: both players benefit when a player eliminates the matching frictions by handling hot potatoes. So, under decentralized matching, there are two types of inefficiencies: for intermediate levels of cost, players may fail to coordinate on the Pareto dominant equilibrium ("coordination failure") and for higher levels of cost, they face a social dilemma. The optimal mechanism eliminates both types of inefficiencies and increases welfare.

However, because the optimal mechanism does not eliminate matching frictions, we also evaluate the welfare loss relative to the no-friction benchmark where players are matched to desirable tasks whenever they become available (while never being matched to undesirable tasks). While the welfare loss due to decentralization vanishes when the cost of handling hot potatoes grows large, this is not the case for the welfare loss due to matching frictions. In fact, it is maximized when the cost of handling hot potatoes is large. The intuition is that inefficiencies due to matching frictions are not due to traditional external effects (which disappear once the private cost dwarfs any social benefit) but instead are driven by indirect effects: When the cost of handling hot potatoes is high, players do not handle them (either in the decentralized or the optimal case) and this aggravates matching frictions.

This paper is organized follows. After a brief literature review, the baseline model is presented in Section 2. Section 3 presents the main results. Section **??** considers variants and extensions of the benchmark model, and Section **??** concludes. Proofs are in the appendix.

1.1 Related literature

Our work contributes to the emerging literature on dynamic matching (e.g., Ünver, 2010; Baccara et al., 2016; Loertscher et al., 2016; Anderson et al., 2017; Doval and Szentes, 2018; Akbarpour et al., 2020).[3] The key tradeoff in this literature – clearing the market now or clearing it later to obtain better matches – is reminiscent of the problem of handling a hot potato now or "kicking the can down the road." However, a crucial distinction is that this literature abstracts from the matching frictions that drive our results. This leads to different conclusions. In the existing literature, thick markets generally improve matching outcomes (as there are more high quality matches available). By contrast, in our setting thickness generally impedes matching (as there are also more low quality matches available) consistent with empirical evidence that thicker markets have lower matching rates due to search frictions (Li and Netessine, 2020).[4] The central importance of matching frictions relates our paper to the literature on search and matching (Burdett and Coles, 1997; Eeckhout, 1999); also see the survey by Rogerson et al. (2005). However, because this literature focuses on two-sided matching problems, the welfare implications are different.[5]

By studying matching in the presence of frictions, our work contributes to the growing literature at the intersection of matching and market design on the one hand and search and matching theory on the other (Chade et al., 2017).[6] As in this literature, indirect effects play a central role in our work: If a player accepts a task then this has an indirect effect on the other player even if the two players do not interact directly, much like in the existing literature a high type taking a low productivity job can affect the payoff to a low ability type. Unlike much of this literature, it studies the wedge between optimal and decentralized matching outcomes and how it depends on key parameters.

Finally, our paper contributes to the literature on stochastic games as the game is a stochastic game where a player's decision whether or not to accept a task today affects the stage game played in the future. Stochastic games were introduced by Shapley (1953) in his seminal work. The literature on stochastic games typically deals general problems

[3] Also see the literature on optimal matching algorithms; e.g., Ashlagi et al. (2018, 2019, 2020) and Hu and Zhou (2018).

[4] Our paper is also distinct from the literature on waiting-list mechanisms (Su and Zenios, 2004, 2005, 2006; Leshno, 2015; Schummer, 2016; Bloch and Cantala, 2017; Thakral, 2019). The key distinction is that, in these models, the availability of undesirable tasks does not make it harder to obtain desirable tasks. These models are therefore unable to capture the key friction considered here.

[5] Specifically, if there are multiple equilibria for our model, they can be Pareto ranked (Proposition 3.1. This is generally not the case for the search and matching literature (Burdett and Coles, 1997).

[6] There is also a recent literature on matching platforms, primarily outside of economics; see, e.g., Kanoria and Saban (2017), Bimpikis et al. (2019), and Arnosti et al. (2020). The issues considered in this literature are largely orthogonal to the questions studied here.

such as the existence of equilibria or characterizing the value (see, e.g., Neyman et al., 2003). By contrast, we provide a full characterization of the Markov perfect equilibria for a particular class of games, and use this to study decentralized and optimal matching protocols.

2 Model

We start by describing the benchmark model. Section **??** considers variants and extensions of the model. Time is discrete and indexed by $t = 0, 1, 2, \ldots$. There are two long-lived players, labeled by $i \in \{1, 2\}$. Each period, a new task arrives. Each task is either a hot potato or a sweet potato, where the probability that the task is a sweet potato equals $\gamma \in (0, 1)$ (independently across tasks and periods). Tasks are short-lived: Each task remains available for two periods.[7] In each period, one of the players is matched to one of the available tasks. Matching is uniform and random. That is, in any period t, each player $i \in \{1, 2\}$ has probability $\frac{1}{2}$ to be matched to one of the available tasks; and all tasks have the same probability of being matched to a player. Once a task is matched to a player, the player decides whether to accept the task or to reject it. The stage payoff (reward), denoted r_i^t, to a player $i \in \{1, 2\}$ equals 0 if it does not accept a task, $u_s > 0$ if it accepts a sweet potato, and $u_h < 0$ if it accepts a hot potato.[8] Players have a common discount factor $\beta \in (0, 1)$. There is no incomplete information: All payoff-relevant parameters (i.e., u_s, u_h, β, γ) are commonly known and actions are publicly observed.

Notice that there are no direct externalities: the stage payoff to a player who is not matched to a task is 0 regardless of the action of the other players. However, there are indirect effects: Because each player's decision whether to accept a task affects the set of tasks that are available in future periods, current actions affect future payoffs. Thus, the game is a stochastic game.

3 Results

3.1 Decentralized matching

Our first main result characterizes the outcome when matching is decentralized. Under decentralized matching, players choose whether to accept or reject a task with the aim of

[7]This assumption is for simplicity. It significantly reduces the number of payoff-relevant states; extending the number of states does not produce substantially new insights.

[8]So, the stage payoff of accepting a task does not depend on how many periods the task has been available. This is not critical for our results: The main rationale is to emphasize that the key dimension on which tasks differ is whether they give a positive or negative stage payoff.

maximizing their discounted sum of payoffs given the other player's strategy.

Some more definitions will be helpful. Players' decisions whether or not to accept depends a task may depend on the available tasks. Because tasks that are not accepted (either because they are rejected or are not assigned to a player) remain available for two periods, there are four *types* of tasks: new sweet potatoes (n,s); new hot potatoes (n,h); old sweet potatoes (o,s); and old hot potatoes (o,h). Let $T := \{(n,s),(n,h),(o,s),(o,h)\}$ be the set of types. Then, $M := T \times \{1,2\}$ is the set of possible *matches* (between players and task types) in a given period. For $i \in \{1,2\}$, let $M_i := T \times \{i\}$ be the set of matches that involve player i. A state specifies the match (i.e., which player is matched to a task and what the type of this task is) as well as whether another task is available, and, if so, what the type of this other task is. That is, let $T^* := T \cup \{\emptyset\}$. Then, a *state* is a tuple $(\tau, i, \tau') \in M \times T^*$. The interpretation is that, if the state is (τ, i, τ'), then player i is matched to a task of type τ, and τ' is the type of the other task that is available if such a task exists, and $\tau' = \emptyset$ otherwise. An *outcome* is state together with an action chosen in that state, i.e., an outcome is of the form $(\tau, i, a, \tau') \in M \times \{0,1\} \times T^*$. Denote the set of outcomes in a given period by $\tilde{H}$. So, for $t > 0$, $\tilde{H}^t$ is the set of all sequences of t outcomes. Let $\tilde{H}^0$ be the null set. Then, a *(behavioral) strategy* for player $i \in \{1,2\}$ is a function

$$\sigma_i : \bigcup_{t=0}^{\infty} (\tilde{H}^{t-1} \times M_i \times T^*) \longrightarrow [0,1].$$

The interpretation is that, for $\tilde{h}^{t-1} \in \tilde{H}^{t-1}$, $(\tau, i) \in M_i$, and $\tau' \in T^*$, $\sigma_i^*(\tilde{h}^{*,t-1}, \tau, i, \tau')$ is the probability that player i accepts a task of type τ following history $\tilde{h}^{*,t-1}$ if i is matched to a task of type τ in t and the other available task is of type $\tau' \in T$ if such a task exists and $\tau' = \emptyset$ otherwise. Let Σ_i be the set of all strategies for player i. As is common, we focus on stationary Markov strategies, that is, strategies that do not depend on the time period or on the entire history of the game, but only on the state. Formally, a strategy σ_i for player $i \in \{1,2\}$ is *stationary Markov* if there exists a function $p_i : M_i \times T^* \to [0,1]$ such that $\sigma_i(\tilde{h}^{t-1}, m_i, \tau') = p_i(m_i, \tau')$ for every time period $t = 0,1,2,\ldots$, history $\tilde{h}^{t-1} \in \tilde{H}^{t-1}$, match $m_i \in M_i$, and $\tau' \in T^*$.

We next define Markov perfect equilibrium. Given any profile $(\sigma_1, \sigma_2) \in \Sigma_1 \times \Sigma_2$ of strategies and the probability $\gamma \in (0,1)$ that a newly arrived task is a sweet potato, let

$$U_i(\sigma_i, \sigma_{-i}) = \mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t\, r_i^t\right]$$

be player i's expected discounted sum of payoffs, where $\mathbb{E}[\cdot]$ is the expectation operator induced by the strategies (σ_1, σ_2) and the probability γ.

Definition 3.1. [**Markov Perfect Equilibrium**] *A Markov Perfect equilibrium is a pair of strategies* $(\sigma_1^*, \sigma_2^*) \in \Sigma_1 \times \Sigma_2$ *such that for each player* $i \in \{1, 2\}$*,* σ_i^* *is stationary Markov and there is no profitable deviation, i.e.,*

$$U_i(\sigma_i^*, \sigma_{i-}^*) \geq U_i(\sigma_i, \sigma_{-i}^*) \quad \textit{for all } \sigma_i \in \Sigma_i.$$

By standard arguments, a Markov perfect equilibrium exists (e.g., Fudenberg and Tirole, 1991, Ch. 13).

To state our characterization result for decentralized matching, we fix a discount factor $\beta \in (0, 1)$, probability $\gamma \in (0, 1)$ that the task is a sweet potato, and payoff parameters $u_s > 0$ and $u_h < 0$. We define $\rho := |u_h/u_s|$ to be the relative cost of accepting a hot potato. Also, define:

$$\rho^1 := \frac{\frac{1}{4}\beta\gamma(1 - \frac{1}{2}\beta)}{1 - \frac{1}{2}\beta\gamma}; \qquad \rho^2 := \frac{\frac{1}{4}\beta\gamma(1 - \frac{1}{2}\beta)}{1 - \frac{1}{4}\beta - \frac{1}{4}\beta\gamma}.$$

Note that $0 < \rho^1 < \rho^2 < 1$. Then:

Proposition 3.1. [**Decentralized Matching**] *In every Markov perfect equilibrium, old hot potatoes are rejected. Sweet potatoes (new or old) are always accepted (conditional on being matched to a player). Moreover,*

(i) if $\rho < \rho^1$*, then players accept new hot potatoes in Markov perfect equilibrium;*

(ii) if $\rho > \rho^2$*, then players reject new hot potatoes in Markov perfect equilibrium;*

(iii) if $\rho^1 < \rho < \rho^2$*, then there are three Markov perfect equilibria. In one equilibrium, players accept new hot potatoes. In the second, players reject new hot potatoes. There is also a mixed equilibrium where players accept new hot potatoes with probability* $q(\beta, \gamma, u_s, u_h) \in (0, 1)$*;*

(iv) in the knife-edge cases $\rho = \rho^1$ *and* $\rho = \rho^2$*, there is a continuum of Markov perfect equilibria. In each equilibrium, one player accepts new hot potatoes with probability 0 or 1, while the other player accepts new hot potatoes with a probability strictly between 0 and 1.*

Proposition 3.1 characterizes the outcomes under decentralized matching where players choose their strategies to maximize their discounted sum of payoffs (taking the strategy of the other player as given). Under decentralized matching, sweet potatoes, both new and old, are accepted. This is intuitive: Handling sweet potatoes is always attractive ($u_s > 0$). Another feature is that old hot potatoes are always rejected. This is also intuitive: There is no reason to accept old hot potatoes because handling hot potatoes is costly ($u_h < 0$)

and old tasks will cease to be available in the next period regardless of whether a player accepts them or not. So, a player's decision whether or not to accept an old hot potato has no impact on which tasks are available in the future. Hence, there is no incentive for players to accept tasks they do not want. Interestingly, players are willing to accept new hot potatoes whenever the cost of handling them is sufficiently small ($\rho \leq \rho^2$). There are two cases. First, when the cost of handling hot potatoes is sufficiently small ($\rho < \rho^1$), then accepting new hot potatoes is the best response for a player regardless of whether the other player accepts or rejects them; hence, in this case, there is a unique Markov perfect equilibrium, and in this equilibrium, both players accept new hot potatoes. Second, for intermediate levels of cost ($\rho \in [\rho^1, \rho^2]$), players' best response depends on the other's strategy: Accepting new hot potatoes is a best response for a player if he believes that the other player accepts them, but not otherwise. Hence, in this case, there are two pure Markov perfect equilibria, one in which players accept new hot potatoes and one in which they reject them. The intuition for why players may wish to accept (new) hot potatoes is that it allows them to eliminate matching frictions, which take the form of players not being matched to sweet potatoes. There are two effects. The first effect is a direct effect: If a given player, say i, rejects a (new) hot potato, then this player is less likely to be matched to a sweet potato in the next period (as he might be matched with the hot potato again). If the relative cost of handling hot potatoes is sufficiently low ($\rho < \rho^1$), this makes it optimal for players to accept (new) hot potatoes even if the other player does not do so. There is also an indirect effect that operates through the actions of the other player: If a given player, say i, rejects a (new) hot potato in a given period t, then the *other* player, say $j \neq i$, is less likely to be matched to a new *hot* potato in $t + 1$ (as this other player might be matched with the old hot potato left over from period t). Now, if j does not accept new hot potatoes, then it does not matter from i's perspective whether j is matched with an old or a new hot potato. Either way, there will be an old hot potato available in $t+2$. But if j *does* accept new hot potatoes, then, if j is matched with a new hot potato, then no old hot potatoes will be available in $t+2$. This increases the chances that i is matched with a sweet potato in $t+2$ (should a sweet potato become available). Thus, if player i rejects a new hot potato in period t, then he is less likely to be matched to a sweet potato in period $t + 2$. But, this is only true if the other player accepts new hot potatoes. This means there are strategic complementarities: If a player accepts new hot potatoes, then the other player has a stronger incentive to do the same. For intermediate levels of costs, this gives rise to equilibrium multiplicity.

One immediate implication of Proposition 3.1 is that players are more willing to accept hot potatoes when they are patient and when the probability that a new task is a sweet potato is high:

Corollary 3.1. [Comparative Statics] *Let u_s, u_h, β, γ be as before. Then,*

(a) For $\beta' < \beta$, if there is a Markov perfect equilibrium in which a player, say i, accepts new hot potatoes (with positive probability) when the discount factor is β', then there is a Markov perfect equilibrium in which i accepts new hot potatoes (with the same probability) when the discount factor is β (assuming that the other parameters u_s, u_h, γ are held fixed).

(b) For $\gamma' < \gamma$, if there is a Markov perfect equilibrium in which a player, say i, accepts new hot potatoes (with positive probability) when the probability of sweet potatoes is γ', then there is a Markov perfect equilibrium in which i accepts new hot potatoes (with the same probability) when the probability of sweet potatoes is γ (assuming that the other parameters u_s, u_h, β are held fixed).

Corollary 3.1 follows directly from the observation that the thresholds ρ^1 and ρ^2 are strictly increasing in β and γ. The result is intuitive: If players are more patient and are more likely to encounter sweet potatoes, then it becomes more important for them to eliminate matching frictions, even if it comes at a short-run cost.

3.2 Optimal matching protocol

We next consider the optimal matching protocol when the designer can only determine players' acceptance decisions but cannot eliminate the matching frictions (i.e., tasks are randomly assigned to agents). A matching protocol or *policy* is a pair $\sigma = (\sigma_1, \sigma_2)$ of strategies. A policy is optimal if it maximizes the sum of discounted aggregate payoff, i.e., σ^{**} is *optimal* if it maximizes

$$\mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t (r_1^t + r_2^t)\right]$$

where the expectation is again taken over outcomes given the policy σ^{**} and the probability γ that a task is a sweet potato. Define

$$\rho^* := \frac{\frac{1}{4}\beta\gamma(2 - \beta(1-\gamma)}{1 - \frac{1}{2}\beta\gamma}.$$

Note that $\rho^2 < \rho^*$. Then:

Proposition 3.2. [Optimal Matching] *Under any optimal policy, old hot potatoes are rejected. Sweet potatoes (new or old) are always accepted (conditional on being matched to a player). Moreover, new hot potatoes are accepted (with probability 1) if $\rho < \rho^*$ whereas they are rejected if $\rho > \rho^*$. In the knife-edge case $\rho = \rho^*$, players either accept or reject new hot potatoes.*

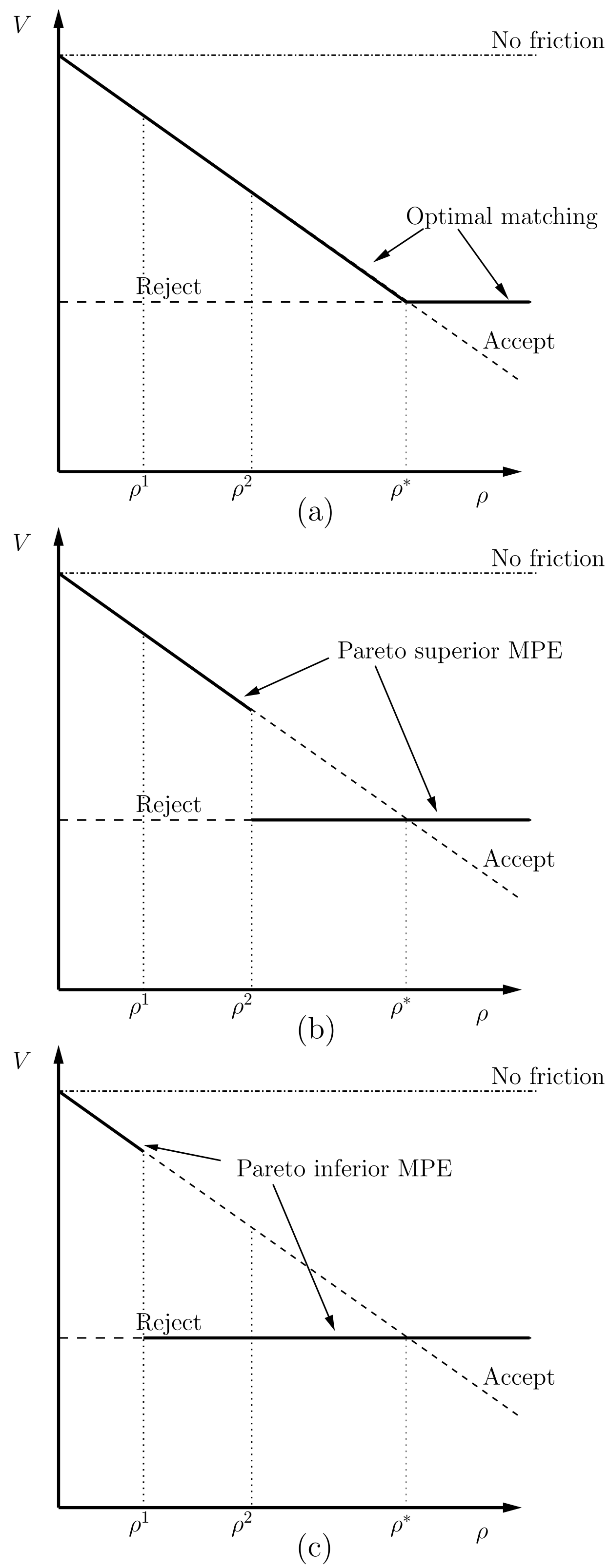

Figure 1: The discounted sum of expected aggregate payoff under (a) the optimal matching protocol; (b) the Pareto-superior Markov perfect equilibrium; and (c) the Pareto-inferior Markov perfect equilibrium, all relative to the no-friction benchmark.

Proposition 3.2 shows that, as under decentralized matching, players accept hot potatoes when the cost of handling them is sufficiently small, and reject them otherwise. However, under the optimal matching policy, players accept new hot potatoes for a larger range of parameters compared to the decentralized case ($\rho^* > \rho^2$). To understand the differences between the optimal matching case in Proposition 3.2 and the decentralized case in Proposition 3.1, consider Figure 1. Figure 1 plots the discounted sum of expected aggregate payoffs as a function of $\rho = |u_h/u_s|$ (for fixed u_s). Figure 1(a) shows the discounted sum $V_{\sigma^{**}}(u_s, u_h, \beta, \gamma)$ of expected aggregate payoffs under the optimal matching policy, and Figures 1(b) and (c) give the discounted sum $V_{\sigma^*}(u_s, u_h, \beta, \gamma)$ of expected aggregate payoffs under decentralized matching for the Pareto-superior and the Pareto-inferior Markov perfect equilibrium, respectively. In each case, players accept new hot potatoes whenever ρ is sufficiently small and reject them otherwise. The discounted sum of expected aggregate payoffs when players accept sweet potatoes and new hot potatoes but reject old hot potatoes is the dashed line labeled "Accept", while the discounted sum of expected aggregate payoffs when players accept sweet potatoes but reject hot potatoes (old or new) is given by the dashed line in each panel labeled "Reject. " Comparing panel (a) to panels (b)–(c) shows that there are two types of inefficiencies under decentralized matching relative to the optimal matching case. First, for higher costs ($\rho \in (\rho^2, \rho^*)$), there is a *social dilemma*: Because players do not internalize the benefit of accepting new hot potatoes for other players, they may reject new hot potatoes under decentralized matching even when they accept them under the optimal matching protocol. This social dilemma arises despite the fact that there are no direct external effects on payoffs. Second, for intermediate costs ($\rho \in (\rho^1, \rho^2)$), there can be *coordination failure* under decentralized matching: Players may fail to select the Pareto-superior equilibrium. Together, these two effects imply that players' payoffs under decentralized matching is significantly below the payoffs under the optimal matching policy whenever the cost of accepting hot potatoes is not too small ($\rho > \rho^1$).

We can also characterize the inefficiency due to matching frictions by comparing the outcome under the optimal matching policy to the no-friction benchmark where the designer removes any hot potatoes and matches players to any sweet potato that becomes available. The discounted sum of expected aggregate payoff under this no-friction benchmark is

$$V_{no\,friction}(u_s, u_h, \beta, \gamma) = \frac{\gamma u_s}{1 - \beta},$$

independent of ρ (for given u_s). This is the dash-dotted line at the top of each panel in Figure 1. Figure 1(a) reveals that the inefficiencies caused by matching frictions are substantial: Even though the optimal matching policies ensures that any externalities are internalized and that there is no coordination failure, the frictions inherent in the matching process make that aggregate payoffs fall short of the no-friction benchmark whenever accepting hot potatoes is costly ($V_{\sigma^{*}*}(u_s, u_h, \beta, \gamma) < V_{no\,friction}(u_s, u_h, \beta, \gamma)$ whenever $\rho > 0$). Moreover, while the difference between the decentralized and optimal matching case vanishes when the cost of handling hot potatoes grows large ($V_{\sigma^{*}*}(u_s, u_h, \beta, \gamma) = V_{\sigma^*}(u_s, u_h, \beta, \gamma)$ for $\rho > \rho^2$), this is not the case for the difference between the optimal matching case and the no-friction benchmark; in fact, the difference $V_{no\,friction}(u_s, u_h, \beta, \gamma) - V_{\sigma^{**}}(u_s, u_h, \beta, \gamma)$ is maximized for $\rho > \rho^2$. Thus, unlike inefficiencies caused by external effects or coordination failure, inefficiencies due to matching frictions remain large even when the cost of accepting hot potatoes is large. Panels (b) and (c) of Figure 1 demonstrate that the combination of inefficiencies due to matching frictions, externalities (panel (b)), and coordination failure (panel (c)) means that players' payoffs under decentralized matching can be significantly below the no-friction benchmark whenever the cost of handling hot potatoes is not too low ($\rho > \rho^1$).

Appendix A Appendix

A.1 Simplifying the state space

Despite the simplicity of the model, the state space is quite extensive (it has 20 states). This complicates the calculations. In this appendix we therefore show that it is without loss of generality to view a player's (stationary Markov) strategy (in the case of decentralized matching) or a policy (in the case of optimal matching) as a function that maps the type of the task that a player is matched to into a (mixed) action for that player. That is, for the purpose of analyzing Markov perfect equilibria or for characterizing the optimal matching protocol, it is without loss of generality to ignore other features of the state, in particular, whether there is another task available, and, if so, what its type is. In addition, we show that, for the purposes of calculating the continuation payoffs, it is possible to work with a "reduced state space" (containing only 6 "reduced states") that does not specify which player is matched to which task. This will simplify the analyses by significantly reducing the set of strategies and policies that need to be considered. Since the arguments for the decentralized and optimal matching case are somewhat different (though closely related), we treat each case separately.

We first show that, for the purposes of analyzing the Markov perfect equilibria, it is without loss of generality to ignore any feature of a state (τ, i, τ') except the match (τ, i). Some more definitions will be helpful. A *reduced outcome* in a given period is a triple (τ, i, a), with $(\tau, i) \in M$ and $a \in \{0, 1\}$, where $(\tau, i, 0)$ means that player i was matched to but rejected a task of type $\tau \in T$ in that period while $(\tau, i, 1)$ means that player i was matched to and accepted a task of type τ. (That is, the difference between an outcome as defined in the main text and a reduced outcome as defined here is that the former specifies whether another task is available, and, if so, what its type is, whereas the latter does not.) Denote the set of possible reduced outcomes by $\tilde{H}^- := M \times \{0,1\}$, and for $t > 0$, let $(\tilde{H}^-)^t$ be the set of all "reduced" t-histories (i.e., a sequence of t reduced outcomes); also, let $(\tilde{H}^-)^0$ be the null set. Then, a *reduced (behavioral) strategy* for player $i \in \{1,2\}$ is a function

$$\sigma_i^- : \bigcup_{t=0}^{\infty} \Big((\tilde{H}^-)^{t-1} \times M_i \Big) \longrightarrow [0,1].$$

The interpretation is again that, for $\tilde{h}^{-,t-1} \in (\tilde{H}^-)^{t-1}$ and $(\tau, i) \in M_i$, $\sigma_i(\tilde{h}^{-,t-1}, i, \tau)$ is the probability that player i accepts a task of type τ following the reduced history $\tilde{h}^{-,t-1}$ (conditional on i being matched to a task of type τ). Let Σ_i^- be the set of all reduced strategies for player i. Clearly, the set of strategies strictly includes the set of reduced strategies. A reduced strategy σ_i for player $i \in \{1,2\}$ is *stationary Markov* if there exists a function $p_i : M_i \to [0,1]$ such that $\sigma_i(\tilde{h}^{-,t-1}, m_i) = p_i(m_i)$ for every time period $t = 0,1,2,\ldots$, reduced history $\tilde{h}^{-,t-1} \in (\tilde{H}^-)^{t-1}$, and match $m_i \in M_i$.

Given any profile $(\sigma_1^-, \sigma_2^-) \in \Sigma_1^- \times \Sigma_2^-$ of reduced strategies and the probability $\gamma \in (0,1)$ that a newly arrived task is a sweet potato, let

$$U_i(\sigma_i, \sigma_{-i}) = \mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t \, r_i^t\right]$$

be player i's expected discounted sum of payoffs, where $\mathbb{E}[\cdot]$ is the expectation operator induced by the strategies (σ_1^-, σ_2^-) and the probability γ. Then, a *reduced Markov Perfect equilibrium* is a pair of reduced strategies $(\sigma_1^*, \sigma_2^*) \in \Sigma_1^- \times \Sigma_2^-$ such that for each player $i \in \{1,2\}$, σ_i^- is stationary Markov and there is no profitable deviation, i.e.,

$$U_i(\sigma_i^*, \sigma_{i-}^*) \geq U_i(\sigma_i, \sigma_{-i}^*) \quad \text{for all } \sigma_i \in \Sigma_i.$$

The following result shows that we can restrict attention to reduced strategies for the purposes of characterizing the Markov equilibria. To formalize this claim, say that a Markov perfect equilibrium and a reduced Markov perfect equilibrium are *equivalent* if for every state (τ, i, τ'), they induce the same probability distribution over actions. Notice that equilibria that are equivalent induce the same expected payoffs for each player. Then, the claim is:

Proposition A.1. [Equivalence MPE and Reduced MPE] *Fix a discount factor $\beta \in (0,1)$, probability $\gamma \in (0,1)$ that a task is a sweet potato, and payoff parameters $u_s > 0$ and $u_h < 0$. Then, the sets of Markov perfect equilibria and reduced Markov* *perfect equilibria are equivalent: For every Markov perfect equilibrium, there is a reduced Markov perfect equilibrium that induces the same action distribution in every state; and for every reduced Markov perfect equilibrium, there is a Markov perfect equilibrium that induces the same action distribution in every state.*

Proof. A reduced Markov perfect equilibrium is characterized by the probabilities $\{q_i^\tau\}_{\tau,i}$ that a player $i \in \{1,2\}$ accepts a task of type $\tau \in T$ when they are matched; likewise, a Markov perfect equilibrium is characterized by the probabilities $\{q_i^\tau(\tau')\}_{\tau,i,\tau'}$ that a player $i \in \{1,2\}$ accepts a task of type $\tau \in T$ when they are matched and the other task is of type $\tau' \in T$ if such a task exists and $\tau' = \emptyset$ otherwise.

Fix a Markov perfect equilibrium defined by the probabilities $\{q_i^\tau(\tau')\}_{\tau,i,\tau'}$. Suppose the current state (at time t) is $(\tau, i, \tau') \in T \times \{1,2\} \times T^*$. We will construct an equivalent Markov perfect equilibrium $\{q_i^\tau\}_{\tau,i}$. First suppose τ is a new task (i.e., $\tau = (n,s)$ or $\tau = (n,h)$). Then, the distribution over available tasks in periods $t' > t$ is independent of τ' (because the other task, if such a task exists, is old and will therefore cease to be available in the next period). Hence, if $q_i^\tau(\tau') \neq q_i^\tau(\tau'')$ for $\tau'' \in T^*$, then this does not affect payoffs (since both current and future payoffs are unaffected by τ', τ''). Therefore, if $q := q_i^\tau(\tau')$ is a best response for player i (given the other player's strategy) when he is matched with a task of type $\tau \in \{(n,s),(n,h)\}$ and the other task has type τ', then q is a best response for player i when he is matched with a task of type τ and the other task has type τ''.[9] Hence, we can set $q_i^\tau := q_i^\tau(\emptyset)$. Next suppose τ is an old task (i.e., $\tau = (o,s)$ or $\tau = (o,h)$). Then, the distribution over tasks available in future periods $t' > t$ does not depend on i's decision whether to accept the task (since the old task will not be available in the next period regardless of i's action). So, while τ' may affect the probability distribution over future states, player i cannot affect this distribution through his action in t. Hence, $q_i^\tau(\tau')$ is chosen only to maximize the stage payoff and this does not depend on τ'. Therefore, if $q := q_i^\tau(\tau')$ is a best response for player i (given the other player's strategy) when he is matched with a task of type τ and the other type has type τ', then q is a best response for player i when he is matched with a task of type τ and the other type has type τ''. Again, we can set $q_i^\tau := q_i^\tau(\emptyset)$. By the above argument, $\{q_i^\tau\}_{\tau,i}$ defines a reduced Markov perfect equilibrium that is equivalent to the Markov perfect equilibrium defined by $\{q_i^\tau(\tau')\}_{\tau,i,\tau'}$.

Conversely, fix a reduced Markov perfect equilibrium characterized by probabilities $\{q_i^\tau\}_{\tau,i}$. We claim that $\{q_i^\tau(\tau')\}_{\tau,i,\tau'}$ with $q_i^\tau(\tau')$ for all $i \in \{1,2\}$, $\tau \in T$ and $\tau' \in T^*$ defines a Markov perfect equilibrium. To see this, suppose that the current state (at time t) is $(\tau, i, \tau') \in T \times \{1,2\} \times T^*$. First suppose τ is a new task (i.e., $\tau = (n,s)$ or $\tau = (n,h)$).

[9] Notice that this argument makes use of the fact that only the player who is matched to a task (here: player i) gets to move in a given period.

Again, the distribution over available tasks in periods $t' > t$ does not depend on τ'. Hence, player i cannot gain by deviating from q_i^τ by conditioning his action on whether $\tau' = (o, s)$, $\tau' = (o, h)$, or $\tau' = \emptyset$. (Recall that these are the only possibilities given that τ is a new task.) So, $q_i^\tau(\tau') = q_i^\tau$ is a best response for player i for any $\tau' \in \{(o, s), (o, h), \emptyset\}$. Next suppose τ is an old task (i.e., $\tau = (o, s)$ or $\tau = (o, h)$). Then, as noted earlier, the distribution over tasks available in future periods $t' > t$ does not depend on i's decision whether or not to accept the task. So, while τ' may affect the probability distribution over future states, player i cannot gain by deviating from q_i^τ by conditioning his action on τ'. Hence, $\{q_i^\tau(\tau')\}_{\tau,i,\tau'}$ with $q_i^\tau(\tau')$ for all $i \in \{1, 2\}$, $\tau \in T$ and $\tau' \in T^*$ defines a Markov perfect equilibrium. $\square$

Proposition A.1 shows that, when considering Markov perfect equilibria, a match (τ, i) is a "sufficient statistic" for the state (τ, i, τ').[10]

We next show that the same result holds for the optimal matching case. Say that a policy (σ_1, σ_2) is a reduced policy if the strategies σ_1, σ_2 are reduced strategies. Then, a reduced policy (σ_1, σ_2) and a policy (σ_1', σ_2') are equivalent if for each player i, the strategies σ_i and σ_i' are equivalent. Again, equivalence in terms of distributions over actions implies equivalence in terms of expected payoffs. Also, a policy (σ_1, σ_2) is stationary Markov if σ_1 and σ_2 are stationary Markov.

Proposition A.2. [Equivalence Stationary Markov Policies] *Fix a discount factor $\beta \in (0, 1)$, probability $\gamma \in (0, 1)$ that a task is a sweet potato, and payoff parameters $u_s > 0$ and $u_h < 0$. Then, the sets of stationary Markov policies and reduced stationary Markov policies are equivalent.*

The proof is essentially identical to that of Proposition A.1 and is therefore omitted.

A.2 Proof of Proposition 3.1

By Proposition A.1 in Appendix A.1, we can restrict attention to reduced Markov perfect equilibria. To ease the presentation, we will drop the qualifier "reduced" in this proof when referring to reduced strategies so to avoid awkward constructions such as stationary Markov reduced strategies. Restricting attention to reduced Markov perfect equilibria significantly simplifies the analysis. To further simplify the problem, it will be convenient

[10] Notice that this is not necessarily true for non-Markovian strategies. For example, if the state at time t is (τ, i, τ') and the other player $j \neq i$ conditions his action in future periods $t' > t$ on the fact that the type of the other task in t was τ' (in addition to other information, such as the state in t'), then it can be optimal for i to condition his action in t on τ'. However, these type of strategies do not seem very natural.

to consider the continuation payoffs a player expects to receive before a match is realized. This will also facilitate the comparison with the optimal matching case (where the identity of the player who is matched to a task is irrelevant). To define the (expected) continuation payoffs, say that the *task composition* specifies the set of the set of available tasks (in a given period). Given that tasks are available for two periods (if they are not accepted by a player), there are six possible task compositions: one (new) sweet potato (denoted by S); one (new) hot potato (H); two sweet potatoes (SS); two hot potatoes (HH); a new hot potato and an old sweet potato (HS); and a new sweet potato and an old hot potato (SH). Let $\Theta := \{S, H, SS, HH, HS, SH\}$ be the set of task compositions. For $\theta \in \Theta$, let $V_i(\theta)$ be the continuation payoff for player $i \in \{1, 2\}$ if the task composition is θ (prior to any match being realized). Let $p_\theta^{\theta'}$ be the transition probability from θ to θ', i.e., probability of reaching $\theta' \in \Theta$ from $\theta \in \Theta$ (given $\{q_i^\tau\}_{\tau \in T, i \in \{1,2\}}$).

Recall that a stationary Markov strategy for a given player is characterized by the probabilities that it accepts a task of each type (conditional on being matched). Let q_i^τ be the probability that player $i \in \{1, 2\}$ accepts a task of type $\tau \in T$ after having been matched with the task.

We first specify the transition probabilities and the continuation payoffs given a profile $\{q_i^\tau\}_{\tau,i}$ of stationary Markov strategies. Note that if the current task composition is S or H, then the task is new (i.e., the type of the task is (n, s) or (n, h), respectively). So, if the current task composition is S, then the transition probabilities are

$$\begin{aligned}
p_S^S &= \gamma(\tfrac{1}{2}q_1^{(n,s)} + \tfrac{1}{2}q_2^{(n,s)});\\
p_S^H &= (1-\gamma)\,(\tfrac{1}{2}q_1^{(n,s)} + \tfrac{1}{2}q_2^{(n,s)});\\
p_S^{SS} &= \gamma(\tfrac{1}{2}(1-q_1^{(n,s)}) + \tfrac{1}{2}(1-q_2^{(n,s)}));\\
p_S^{HS} &= (1-\gamma)\,(\tfrac{1}{2}(1-q_1^{(n,s)}) + \tfrac{1}{2}(1-q_2^{(n,s)}));
\end{aligned}$$

and the expected continuation payoff is

$$V_i(S) = \tfrac{1}{2}q_i^{(n,s)}u_s + \beta\left[p_S^S V_i(S) + p_S^H V_i(H) + p_S^{SS} V_i(SS) + p_S^{HS} V_i(HS)\right]. \tag{A.1}$$

Second, if the current task composition is H, then the transition probabilities are

$$\begin{aligned}
p_H^S &= \gamma(\tfrac{1}{2}q_1^{(n,h)} + \tfrac{1}{2}q_2^{(n,h)});\\
p_H^H &= (1-\gamma)\,(\tfrac{1}{2}q_1^{(n,h)} + \tfrac{1}{2}q_2^{(n,h)});\\
p_H^{SH} &= \gamma(\tfrac{1}{2}(1-q_1^{(n,h)}) + \tfrac{1}{2}(1-q_2^{(n,h)}));\\
p_H^{HH} &= (1-\gamma)\,(\tfrac{1}{2}(1-q_1^{(n,h)}) + \tfrac{1}{2}(1-q_2^{(n,h)}));
\end{aligned}$$

and the expected continuation payoff is

$$V_i(H) = \tfrac{1}{2}q_i^{(n,h)}u_h + \beta\left[p_H^S V_i(S) + p_H^H V_i(H) + p_H^{SH} V_i(SH) + p_H^{HH} V_i(HH)\right]. \tag{A.2}$$

Third, if the current task composition is SS, then the transition probabilities are

$$\begin{aligned}
p_{SS}^{S} &= \tfrac{1}{2}p_{S}^{S};\\
p_{SS}^{H} &= \tfrac{1}{2}p_{S}^{H};\\
p_{SS}^{SS} &= \tfrac{1}{2}\gamma + \tfrac{1}{2}p_{S}^{SS};\\
p_{SS}^{HS} &= \tfrac{1}{2}(1-\gamma) + \tfrac{1}{2}p_{S}^{HS};
\end{aligned}$$

and the expected continuation payoff is (using (A.1)),

$$V_i(SS) = \tfrac{1}{4}q_i^{(o,s)}u_s + \tfrac{1}{2}V_i(S) + \tfrac{1}{2}\beta\left[\gamma V_i(SS) + (1-\gamma)V_i(HS)\right]. \tag{A.3}$$

Fourth, if the current task composition is SH, then the transition probabilities are

$$\begin{aligned}
p_{SH}^{S} &= p_{SS}^{S};\\
p_{SH}^{H} &= p_{SS}^{H};\\
p_{SH}^{SS} &= p_{SS}^{SS};\\
p_{SH}^{HS} &= p_{SS}^{HS};
\end{aligned}$$

and the expected continuation payoff satisfies

$$V_i(SS) - V_i(SH) = \tfrac{1}{4}q_i^{(o,s)}u_s - \tfrac{1}{4}q_i^{(o,h)}u_h. \tag{A.4}$$

Fifth, if the current task composition is HH, then the transition probabilities are

$$\begin{aligned}
p_{HH}^{S} &= \tfrac{1}{2}p_{H}^{S};\\
p_{HH}^{H} &= \tfrac{1}{2}p_{H}^{H};\\
p_{HH}^{SH} &= \tfrac{1}{2}\gamma + \tfrac{1}{2}p_{H}^{SH};\\
p_{HH}^{HH} &= \tfrac{1}{2}(1-\gamma) + \tfrac{1}{2}p_{H}^{HH}.
\end{aligned}$$

and the expected continuation payoff is (using (A.2)),

$$V_i(HH) = \tfrac{1}{4}q_i^{(o,h)}u_h + \tfrac{1}{2}V_i(H) + \tfrac{1}{2}\beta\left[\gamma V_i(SH) + (1-\gamma)V_i(HH)\right]. \tag{A.5}$$

Sixth, if the current task composition is HS, then the transition probabilities are

$$\begin{aligned}
p_{HS}^{S} &= p_{HH}^{S};\\
p_{HS}^{H} &= p_{HH}^{H};\\
p_{HS}^{SH} &= p_{HH}^{SH};\\
p_{HS}^{HH} &= p_{HH}^{HH};
\end{aligned}$$

and the expected continuation payoff satisfies

$$V_i(HS) - V_i(HH) = \tfrac{1}{4} q_i^{(o,s)} u_s - \tfrac{1}{4} q_i^{(o,h)} u_h. \tag{A.6}$$

Also, note that by the one-shot deviation principle, a stationary Markov strategy $\{q_i^\tau\}_\tau$ for a player $i \in \{1,2\}$ is a best response to a stationary Markov strategy $\{q_j^\tau\}_\tau$ for player $j \neq i$ if and only if there is no profitable one-shot deviation in any $\theta \in \Theta$ where player i is matched with type τ (assuming that the continuation payoffs are determined by the probabilities $\{q_1^\tau\}_\tau$ and $\{q_2^\tau\}_\tau$).

The proof now follows from a series of lemmas. The first results characterizes a player's best response given that the other player plays a reduced stationary Markov strategy.

Lemma A.1. *Given a reduced stationary Markov strategy for player $j \in \{1,2\}$, a reduced stationary Markov strategy $\{q_i^\tau\}_\tau$ is a best response for player $i \in \{1,2\}$, $i \neq j$, if and only if the following four conditions hold: (1) $q_i^{(o,s)} = 1$; (2) $q_i^{(o,h)} = 0$; (3) one of the following holds:*

$$\begin{aligned}
u_s + \beta\left[\gamma V_i(S) + (1-\gamma)V_i(H)\right] &\geq \beta\left[\gamma V_i(SS) + (1-\gamma)V_i(HS)\right], \quad q_i^{(n,s)} = 1;\\
u_s + \beta\left[\gamma V_i(S) + (1-\gamma)V_i(H)\right] &= \beta\left[\gamma V_i(SS) + (1-\gamma)V_i(HS)\right], \quad q_i^{(n,s)} \in [0,1];\\
u_s + \beta\left[\gamma V_i(S) + (1-\gamma)V_i(H)\right] &\leq \beta\left[\gamma V_i(SS) + (1-\gamma)V_i(HS)\right], \quad q_i^{(n,s)} = 0;
\end{aligned}$$

and (4) one of the following holds:

$$\begin{aligned}
u_h + \beta\left[\gamma V_i(S) + (1-\gamma)V_i(H)\right] &\geq \beta\left[\gamma V_i(SH) + (1-\gamma)V_i(HH)\right] \quad q_i^{(n,h)} = 1,\\
u_h + \beta\left[\gamma V_i(S) + (1-\gamma)V_i(H)\right] &= \beta\left[\gamma V_i(SH) + (1-\gamma)V_i(HH)\right] \quad q_i^{(n,h)} \in [0,1],\\
u_h + \beta\left[\gamma V_i(S) + (1-\gamma)V_i(H)\right] &\leq \beta\left[\gamma V_i(SH) + (1-\gamma)V_i(HH)\right] \quad q_i^{(n,h)} = 0.
\end{aligned}$$

Proof. We start with (1). The result is immediate: For any $\theta \in \Theta$ where player i may be matched with an old sweet potato, accepting the old sweet potato gives a positive payoff $u_s > 0$ in the current period and does not influence the transition probabilities (i.e., $p_\theta^{\theta'}$ is independent of $q_i^{(o,s)}$ for all $i \in \{1,2\}$, $\theta, \theta' \in \Theta$). So, there is no profitable one-shot deviation if and only if $q_i^{(o,s)} = 1$. Next consider (2). The result is again immediate: For any $\theta \in \Theta$ where player i can be matched with an old hot potato, accepting the old hot potato gives a negative payoff $u_h < 0$ in the current period and does not influence the transition probabilities (i.e., $p_\theta^{\theta'}$ is independent of $q_i^{(o,h)}$ for all $i \in \{1,2\}$, $\theta, \theta' \in \Theta$). So, there is no profitable one-shot deviation if and only if $q_i^{(o,h)} = 0$.

We next consider (3). We derive these conditions from the incentive compatibility constraints for player i when he is matched with type $\tau = (n,s)$. First suppose $q_i^{(n,s)} = 1$. There is no profitable deviation for player i when he is matched with (n,s) if and only if

$$u_s + \beta\left[\gamma V_i(S) + (1-\gamma)V_i(H)\right] \geq \beta\left[\gamma V_i(SS) + (1-\gamma)V_i(HS)\right].$$

Next suppose $q_i^{(n,s)} \in [0,1]$. Then there is no profitable deviation for player i when he is matched with (n,s) in if and only if

$$u_s + \beta\left[\gamma V_i(S) + (1-\gamma)V_i(H)\right] = \beta\left[\gamma V_i(SS) + (1-\gamma)V_i(HS)\right].$$

Finally, suppose $q_i^{(n,s)} = 0$. Then there is no profitable deviation for player i when he is matched with (n,s) if and only if

$$u_s + \beta\left[\gamma V_i(S) + (1-\gamma)V_i(H)\right] \leq \beta\left[\gamma V_i(SS) + (1-\gamma)V_i(HS)\right].$$

It remains to consider (4). We derive these conditions from the incentive compatibility constraints for player i when he is matched with type $\tau = (n,h)$. First suppose $q_i^{(n,h)} = 1$. There is no profitable deviation for player i when he is matched with (n,h) if and only if

$$u_h + \beta\left[\gamma V_i(S) + (1-\gamma)V_i(H)\right] \geq \beta\left[\gamma V_i(SH) + (1-\gamma)V_i(HH)\right].$$

Next suppose $q_i^{(n,h)} \in [0,1]$. Then there is no profitable deviation for player i when he is matched with (n,h) if and only if

$$u_h + \beta\left[\gamma V_i(S) + (1-\gamma)V_i(H)\right] = \beta\left[\gamma V_i(SH) + (1-\gamma)V_i(HH)\right].$$

Finally, suppose $q_i^{(n,h)} = 0$. Then there is no profitable deviation for player i when he is matched with (n,h) if and only if

$$u_h + \beta\left[\gamma V_i(S) + (1-\gamma)V_i(H)\right] \leq \beta\left[\gamma V_i(SH) + (1-\gamma)V_i(HH)\right].$$

This proves the lemma. $\square$

The result says that in any reduced Markov perfect equilibrium (and thus in any Markov perfect equilibrium), old tasks are accepted if they are sweet potatoes but not if they are hot potatoes. This is intuitive. Suppose that a player is matched with an old task in a given period t. Then the future payoffs are independent of the player's decision whether or not to accept this task. This follows because, in either case, the old task is no longer available in $t+1$ and the new task from t becomes the old task in $t+1$. Thus, if a player is matched to an old task, the unique best response is to accept the old task if it is a sweet potato and not to accept it if he is a hot potato.

Lemma A.1 and Eqs. (A.3)–(A.6) imply

$$\left(\gamma V_i(SS) + (1-\gamma)V_i(HS)\right)\left(1 - \tfrac{1}{2}\beta\right) = \tfrac{1}{4}u_s\left(1 - \tfrac{1}{2}\beta(1-\gamma)\right) + \tfrac{1}{2}\left[\gamma V_i(S) + (1-\gamma)V_i(H)\right] \quad \text{(A.7)}$$

and

$$\left(\gamma V_i(SH) + (1-\gamma)V_i(HH)\right)\left(1 - \tfrac{1}{2}\beta\right) = \tfrac{1}{8}\beta\gamma u_s + \tfrac{1}{2}\left[\gamma V_i(S) + (1-\gamma)V_i(H)\right]. \qquad \text{(A.8)}$$

The following lemma shows that players accept new sweet potatoes in any (reduced) Markov perfect equilibrium.

Lemma A.2. *In any reduced Markov perfect equilibrium,* $q_i^{(n,s)} = 1$ *for* $i \in \{1,2\}$.

Proof. Consider the reduced strategy that accepts only sweet potatoes (new or old). This reduced strategy strategy yields a positive payoff in every period that the player is matched, regardless of the strategy of the other player (stationary Markov or not). Hence, in any Markov perfect equilibrium, the expected discounted payoff for player i must be positive. In particular, if $\theta = S$,

$$\max\left\{u_s + \beta\left[\gamma V_i(S) + (1-\gamma)V_i(H)\right], \beta\left[\gamma V_i(SS) + (1-\gamma)V_i(HS)\right]\right\} > 0.$$

Thus, if $\gamma V_i(SS) + (1-\gamma)V_i(HS) \leq 0$ then

$$u_s + \beta\left[\gamma V_i(S) + (1-\gamma)V_i(H)\right] > \beta\left[\gamma V_i(SS) + (1-\gamma)V_i(HS)\right],$$

and the result follows from Lemma A.1. So suppose $\gamma V_i(SS) + (1-\gamma)V_i(HS) > 0$. By (A.7),

$$\begin{aligned}(\gamma V_i(S) + (1-\gamma)V_i(H)) - (\gamma V_i(SS) + (1-\gamma)V_i(HS)) = \\ \left[\gamma V_i(SS) + (1-\gamma)V_i(HS)\right](1-\beta) - \tfrac{1}{2}u_s(1 - \tfrac{1}{2}\beta(1-\gamma)).\end{aligned}$$

Hence,

$$u_s + \beta\left[\gamma V_i(S) + (1-\gamma)V_i(H)\right] > \beta\left[\gamma V_i(SS) + (1-\gamma)V_i(HS)\right].$$

The conclusion now follows from Lemma A.1. □

Lemma A.3. *Suppose that player* $j \in \{1,2\}$ *accepts any sweet potato and any new hot potato, but that he does not accept any old hot potatoes (i.e.,* $q_j^{(n,s)} = q_j^{(o,s)} = q_j^{(n,h)} = 1$ *and* $q_j^{(o,h)} = 0$*). Then, the same reduced stationary Markov strategy (i.e.,* $q_i^{(n,s)} = q_i^{(o,s)} = q_i^{(n,h)} = 1$*, and* $q_i^{(o,h)} = 0$*) is a best response for player* $i \neq j$ *if and only if*

$$\left|\frac{u_h}{u_s}\right| \leq \frac{\frac{1}{4}\beta\gamma\left(1 - \frac{1}{2}\beta\right)}{1 - \frac{1}{4}\beta - \frac{1}{4}\beta\gamma}.$$

Proof. By (A.1) and (A.2),

$$\begin{aligned}V_i(S) &= \tfrac{1}{2}u_s + \beta\left[p_S^S V_i(S) + p_S^H V_i(H) + p_S^{SS} V_i(SS) + p_S^{HS} V_i(HS)\right];\\ V_i(H) &= \tfrac{1}{2}u_h + \beta\left[p_H^S V_i(S) + p_H^H V_i(H) + p_H^{SH} V_i(SH) + p_H^{HH} V_i(HH)\right];\end{aligned}$$

moreover,

$$\begin{array}{llll} p_S^S = \gamma; & p_S^H = 1-\gamma; & p_S^{SS} = 0; & p_S^{HS} = 0;\\ p_H^S = \gamma; & p_H^H = 1-\gamma; & p_H^{SH} = 0; & p_H^{HH} = 0.\end{array}$$

Therefore,

$$\gamma V_i(S) + (1-\gamma)V_i(H) = \frac{\frac{1}{2}\left[\gamma u_s + (1-\gamma)u_h\right]}{1-\beta}.$$

By (A.8),

$$\begin{aligned}\left[\gamma V_i(SH) + (1-\gamma)V_i(HH)\right]\left(1-\tfrac{1}{2}\beta\right) &= \tfrac{1}{8}\beta\gamma u_s + \tfrac{1}{2}\left(\gamma V_i(S) + (1-\gamma)V_i(H)\right)\\ &= \tfrac{1}{8}\beta\gamma u_s + \frac{\frac{1}{4}\left(\gamma u_s + (1-\gamma)u_h\right)}{1-\beta}.\end{aligned}$$

Thus,

$$\begin{aligned}&\left(\gamma V_i(S) + (1-\gamma)V_i(H)\right) - \left(\gamma V_i(SH) + (1-\gamma)V_i(HH)\right)\\ &\quad= \frac{\frac{1}{2}\left(\gamma u_s + (1-\gamma)u_h\right)}{1-\beta} - \frac{\frac{1}{8}\beta\gamma u_s}{1-\frac{1}{2}\beta} - \frac{\frac{1}{4}\left(\gamma u_s + (1-\gamma)u_h\right)}{(1-\beta)\left(1-\frac{1}{2}\beta\right)}\\ &\quad= \frac{\frac{1}{4}\left(\gamma u_s + (1-\gamma)u_h\right) - \frac{1}{8}\beta\gamma u_s}{1-\frac{1}{2}\beta}\end{aligned}$$

Hence,

$$u_h + \beta\Big[\left(\gamma V_i(S) + (1-\gamma)V_i(H)\right) - \left(\gamma V_i(SH) + (1-\gamma)V_i(HH)\right)\Big] \geq 0$$

if and only if

$$u_h + \beta\left(\frac{\frac{1}{4}\left(\gamma u_s + (1-\gamma)u_h\right) - \frac{1}{8}\beta\gamma u_s}{1-\frac{1}{2}\beta}\right) \geq 0$$

and, therefore, if and only if

$$u_h \geq -\frac{\frac{1}{4}\beta\gamma\left(1-\frac{1}{2}\beta\right)}{1-\frac{1}{2}\beta+\frac{1}{4}\beta(1-\gamma)}u_s.$$

Finally, recall that by (A.4) and (A.6)

$$\begin{aligned}V_i(SS) - V_i(SH) &= \tfrac{1}{4}u_s;\\ V_i(HS) - V_i(HH) &= \tfrac{1}{4}u_s.\end{aligned}$$

Therefore,

$$
\begin{aligned}
&[\gamma V_i(S) + (1-\gamma)V_i(H)] - [\gamma V_i(SS) + (1-\gamma)\, V_i(HS)] \\
&\quad = [\gamma V_i(S) + (1-\gamma)V_i(H)] - [\gamma V_i(SH) + (1-\gamma)\, V_i(HH)] - \tfrac{1}{4}u_s \\
&\quad = \frac{\frac{1}{4}\left[\gamma u_s + (1-\gamma)u_h\right]}{1-\frac{1}{2}\beta} - \frac{\frac{1}{8}\beta\gamma u_s}{1-\frac{1}{2}\beta} - \tfrac{1}{4}u_s.
\end{aligned}
$$

Consequently,

$$
u_s + \beta\Big[(\gamma V_i(S) + (1-\gamma)V_i(H)) - (\gamma V_i(SS) + (1-\gamma)V_i(HS))\Big] \geq 0.
$$

The conclusion now follows from Lemma A.1. □

Lemma A.4. *Suppose that player $j \in \{1,2\}$ accepts any sweet potato but rejects any hot potato (i.e., $q_j^{(n,s)} = q_j^{(o,s)} = 1$ and $q_j^{(n,h)} = q_j^{(o,h)} = 0$). Then, the same reduced stationary Markov strategy (i.e., $q_i^{(n,s)} = q_i^{(o,s)} = 1$ and $q_i^{(n,h)} = q_i^{(o,h)} = 0$) is a best response for player $i \neq j$ if and only if*

$$
\left|\frac{u_h}{u_s}\right| \geq \frac{\frac{1}{4}\beta\gamma\left(1-\frac{1}{2}\beta\right)}{1-\frac{1}{2}\beta\gamma}.
$$

Proof. By (A.1) and (A.2),

$$
\begin{aligned}
V_i(S) &= \tfrac{1}{2}u_s + \beta\left[p_S^S V_i(S) + p_S^H V_i(H) + p_S^{SS} V_i(SS) + p_S^{HS} V_i(HS)\right]; \\
V_i(H) &= \beta\left[p_H^S V_i(S) + p_H^H V_i(H) + p_H^{SH} V_i(SH) + p_H^{HH} V_i(HH)\right].
\end{aligned}
$$

moreover,

$$
\begin{array}{llll}
p_S^S = \gamma; & p_S^H = 1-\gamma; & p_S^{SS} = 0; & p_S^{HS} = 0; \\
p_H^S = 0; & p_H^H = 0; & p_H^{SH} = \gamma; & p_H^{HH} = 1-\gamma.
\end{array}
$$

Thus,

$$
\begin{aligned}
V_i(S) &= \tfrac{1}{2}u_s + \beta\big(\gamma V_i(S) + (1-\gamma)\, V_i(H)\big) \\
V_i(H) &= \beta\left[\gamma V_i(SH) + (1-\gamma)\, V_i(HH)\right].
\end{aligned}
$$

Consequently,

$$
(\gamma V_i(S) + (1-\gamma)V_i(H))\,(1-\beta\gamma) = \tfrac{1}{2}\gamma u_s + \beta\,(1-\gamma)\left[\gamma V_i(SH) + (1-\gamma)\, V_i(HH)\right].
$$

By (A.8),

$$
\tfrac{1}{2}\left(\gamma V_i(S) + (1-\gamma)V_i(H)\right) = -\tfrac{1}{8}\beta\gamma u_s + \left(1-\tfrac{1}{2}\beta\right)\left(\gamma V_i(SH) + (1-\gamma)V_i(HH)\right).
$$

Solving this linear system of two equations for the terms $(\gamma V_i(S) + (1-\gamma)V_i(H))$ and $\left(1-\frac{1}{2}\beta\right)(\gamma V_i(SH) + (1-\gamma)V_i(HH))$ and combining the results yields

$$[\gamma V_i(S) + (1-\gamma)V_i(H)] - [\gamma V_i(SH) + (1-\gamma)\, V_i(HH)] = \frac{\frac{1}{4}\gamma u_s(1-\frac{1}{2}\beta)}{1-\frac{1}{2}\beta\gamma}.$$

Hence,

$$u_h + \beta\left([\gamma V_i(S) + (1-\gamma)V_i(H)] - [\gamma V_i(SH) + (1-\gamma)\, V_i(HH)]\right) \leq 0$$

if and only if

$$u_h + \beta\left(\frac{\frac{1}{4}\gamma u_s(1-\frac{1}{2}\beta)}{1-\frac{1}{2}\beta\gamma}\right) \leq 0,$$

or, equivalently,

$$u_h \leq -\frac{\frac{1}{4}\beta\gamma u_s(1-\frac{1}{2}\beta)}{1-\frac{1}{2}\beta\gamma}.$$

Finally, by (A.4) and (A.6),

$$\begin{aligned} V_i(SS) - V_i(SH) &= \tfrac{1}{4}u_s; \\ V_i(HS) - V_i(HH) &= \tfrac{1}{4}u_s. \end{aligned}$$

Combining these results,

$$[\gamma V_i(S) + (1-\gamma)V_i(H)] - [\gamma V_i(SS) + (1-\gamma)\, V_i(HS)] = \frac{\frac{1}{4}\gamma u_s(1-\frac{1}{2}\beta)}{1-\frac{1}{2}\beta\gamma} - \tfrac{1}{4}u_s.$$

Thus,

$$u_s + \beta\left([\gamma V_i(S) + (1-\gamma)V_i(H)] - [\gamma V_i(SS) + (1-\gamma)V_i(HS)]\right) \geq 0.$$

The conclusion now follows from Lemma A.1. □

Lemma A.5. *If*

$$-\frac{\frac{1}{4}\beta\gamma\left(1-\frac{1}{2}\beta\right)}{1-\frac{1}{4}\beta-\frac{1}{4}\beta\gamma} \geq \left|\frac{u_h}{u_s}\right| \geq -\frac{\frac{1}{4}\beta\gamma\left(1-\frac{1}{2}\beta\right)}{1-\frac{1}{2}\beta\gamma}. \tag{A.9}$$

then there is a reduced Markov perfect equilibrium such that each player $i \in \{1,2\}$ accepts new hot potatoes with probability

$$q_i^{(n,h)} = \frac{u_h\left(1-\frac{1}{2}\beta\gamma\right) + \frac{1}{4}\beta\gamma u_s - \frac{1}{8}\beta^2\gamma u_s}{\frac{1}{4}\beta u_h\,(1-\gamma)}. \tag{A.10}$$

There are no other reduced Markov perfect equilibria with $q_i^{(n,h)} \in (0,1)$ for each $i \in \{1,2\}$. There is also no reduced Markov perfect equilibrium with $q_i^{(n,h)} \in (0,1)$ for some player

$i \in \{1,2\}$ and $q_j^{(n,h)} \in \{0,1\}$ for $j \neq i$, except in knife-edge cases where $\rho = \frac{\frac{1}{4}\beta\gamma\left(1-\frac{1}{2}\beta\right)}{1-\frac{1}{2}\beta\gamma}$ or $\rho = \frac{\frac{1}{4}\beta\gamma\left(1-\frac{1}{2}\beta\right)}{1-\frac{1}{4}\beta-\frac{1}{4}\beta\gamma}$, in which case $q_j^{(n,h)} = 1$, $q_i^{(n,h)} \in [0,1]$ for $i \neq j$.

Proof. If $q_1^{(n,s)} = q_2^{(n,s)} = 1$,

$$p_S^S = \gamma; \qquad p_S^H = 1-\gamma;$$
$$p_S^{SS} = 0; \qquad p_S^{HS} = 0.$$

Thus,

$$V_i(S) = \tfrac{1}{2}u_s + \beta\left[\gamma V_i(S) + (1-\gamma)\, V_i(H)\right]$$

In addition, let $q_1^{(n,h)} + q_2^{(n,h)} = q$. Then,

$$p_H^S = \tfrac{1}{2}\gamma q; \qquad p_H^H = \tfrac{1}{2}(1-\gamma)q;$$
$$p_H^{SH} = \gamma(1-\tfrac{1}{2}q); \qquad p_H^{HH} = (1-\gamma)\,(1-\tfrac{1}{2}q).$$

Hence,

$$V_i(H) = \tfrac{1}{2}q_i^{(n,h)}u_h + \tfrac{1}{2}\beta q\left[\gamma V_i(S) + (1-\gamma)\, V_i(H)\right] + \beta(1-\tfrac{1}{2}q)\left[\gamma V_i(SH) + (1-\gamma)\, V_i(HH)\right]$$

and

$$(\gamma V_i(S) + (1-\gamma)\, V_i(H))(1-\beta\gamma-\tfrac{1}{2}\beta q\,(1-\gamma)) = \\ \tfrac{1}{2}\gamma u_s + \tfrac{1}{2}\,(1-\gamma)\, q_i^{(n,h)}u_h + \beta\,(1-\gamma)\,(1-\tfrac{1}{2}q)\left[\gamma V_i(SH) + (1-\gamma)\, V_i(HH)\right].$$

By (A.8),

$$\left[\gamma V_i(SH) + (1-\gamma)V_i(HH)\right]\left(1-\tfrac{1}{2}\beta\right) = \tfrac{1}{8}\beta\gamma u_s + \tfrac{1}{2}\left[\gamma V_i(S) + (1-\gamma)V_i(H)\right].$$

Again, solving this system of linear equations for $\gamma V_i(S) + (1-\gamma)\, V_i(H)$ and $\gamma V_i(SH) + (1-\gamma)\, V_i(HH)$ and rearranging gives

$$\left[\gamma V_i(S) + (1-\gamma)\, V_i(H)\right] - \left[\gamma V_i(SH) + (1-\gamma)\, V_i(HH)\right] = \\ \frac{\frac{1}{4}\gamma u_s + \frac{1}{4}\,(1-\gamma)\, q_i^{(n,h)}u_h - \frac{1}{8}\beta\gamma u_s}{1-\frac{1}{2}\beta\gamma-\frac{1}{4}\beta q\,(1-\gamma)}.$$

By Lemma A.1, both players are indifferent between accepting and rejecting new hot potatoes if and only if for $i \in \{1,2\}$,

$$u_h + \beta\left(\left[\gamma V_i(S) + (1-\gamma)\, V_i(H)\right] - \left[\gamma V_i(SH) + (1-\gamma)\, V_i(HH)\right]\right) = 0. \qquad \text{(A.11)}$$

By the above argument, this holds if and only if

$$u_h + \beta \left(\frac{\frac{1}{4}\gamma u_s + \frac{1}{4}(1-\gamma) q_i^{(n,h)} u_h - \frac{1}{8}\beta\gamma u_s}{1 - \frac{1}{2}\beta\gamma - \frac{1}{4}\beta q (1-\gamma)} \right) = 0.$$

This holds if and only if $q_1^{(n,h)} = q_2^{(n,h)} = \frac{1}{2} q$ and

$$u_h \left(1 - \tfrac{1}{2}\beta\gamma - \tfrac{1}{4}\beta q (1-\gamma)\right) = -\beta \left(\tfrac{1}{4}\gamma u_s + \tfrac{1}{4}(1-\gamma) q u_h - \tfrac{1}{8}\beta\gamma u_s\right).$$

The latter is equivalent to

$$q = \frac{u_h \left(1 - \frac{1}{2}\beta\gamma\right) + \frac{1}{4}\beta\gamma u_s - \frac{1}{8}\beta^2\gamma u_s}{\frac{1}{8}\beta(1-\gamma) u_h}.$$

It will be convenient to introduce the notation $\bar{q} := q$; we have $\bar{q} \in [0,2]$ if and only if

$$\frac{\frac{1}{4}\beta\gamma\left(1 - \frac{1}{2}\beta\right)}{1 - \frac{1}{2}\beta - \frac{1}{4}\beta\gamma} \geq \left|\frac{u_h}{u_s}\right| \geq \frac{\frac{1}{4}\beta\gamma\left(1 - \frac{1}{2}\beta\right)}{1 - \frac{1}{2}\beta\gamma}. \tag{A.12}$$

Again, by (A.4) and (A.6),

$$\begin{aligned} V_i(SS) - V_i(SH) &= \tfrac{1}{4} u_s; \\ V_i(HS) - V_i(HH) &= \tfrac{1}{4} u_s. \end{aligned}$$

Therefore, if (A.11) holds, then

$$u_s + \beta \Big([\gamma V_i(S) + (1-\gamma) V_i(H)] - [\gamma V_i(SS) + (1-\gamma) V_i(HS)] \Big) \geq 0.$$

By Lemma A.1, if (A.12) holds, then there is a reduced Markov perfect equilibrium where $q_i^{(o,s)} = q_i^{(n,s)} = 1$, $q_i^{(o,h)} = 0$, $q_i^{(n,h)} = \frac{1}{2}\bar{q}$ for $i \in \{1,2\}$. There is no other reduced Markov perfect equilibrium with $q_i^{(n,h)} \in (0,1)$ for $i \in \{1,2\}$. This proves the first claim.

Now assume that one player rejects hot potatoes while the other player randomizes over accepting and rejecting new hot potatoes. That is, $q_j^{(n,h)} = 0$ and $q_i^{(n,h)} \in (0,1)$ (where $j \neq i$). Then, by previous results, $q_i^{(o,s)} = q_i^{(n,s)} = 1$, $q_i^{(o,h)} = 0$, and $q_i^{(n,h)} = q$. So, the indifference condition for player i becomes

$$u_h + \beta \left(\frac{\frac{1}{4}\gamma u_s + \frac{1}{4}(1-\gamma) q u_h - \frac{1}{8}\beta\gamma u_s}{1 - \frac{1}{2}\beta\gamma - \frac{1}{4}\beta q (1-\gamma)} \right) = 0.$$

This is equivalent to

$$u_h = -\frac{\frac{1}{4}\beta\gamma u_s \left(1 - \frac{1}{2}\beta\right)}{1 - \frac{1}{2}\beta\gamma}.$$

Next assume that one player accepts hot potatoes while the other player randomizes over accepting and rejecting new hot potatoes. That is, $q_j^{(n,h)} = 1$ and $q_i^{(n,h)} \in (0,1)$. Then, $q_i^{(n,h)} = q - 1$ and for each $i \in \{1,2\}$, $q_i^{(o,s)} = q_i^{(n,s)} = 1$, $q_i^{(o,h)} = 0$; So, the indifference condition for player i becomes

$$u_h + \beta \left(\frac{\frac{1}{4}\gamma u_s + \frac{1}{4}(1-\gamma)(1+q) u_h - \frac{1}{8}\beta\gamma u_s}{1 - \frac{1}{2}\beta\gamma - \frac{1}{4}\beta q (1-\gamma)} \right) = 0,$$

or, equivalently,

$$u_h = -\frac{\frac{1}{4}\beta\gamma u_s \left(1 - \frac{1}{2}\beta\right)}{1 - \frac{1}{4}\beta - \frac{1}{4}\beta\gamma}.$$

This proves the lemma. □

We are now ready to prove Proposition 3.1. Let

$$\rho^1 := \frac{\frac{1}{4}\beta\gamma\left(1 - \frac{1}{2}\beta\right)}{1 - \frac{1}{2}\beta\gamma} \text{ and } \rho^2 := \frac{\frac{1}{4}\beta\gamma(1 - \frac{1}{2}\beta)}{1 - \frac{1}{4}\beta - \frac{1}{4}\beta\gamma}$$

and note that $\rho^1 < \rho^2$. Also recall that $\rho := |u_h/u_s|$. By Lemmas A.1–A.2, in any reduced Markov perfect equilibrium (and therefore in any Markov perfect equilibrium), players reject old hot potatoes while they accept sweet potatoes (new or old). By Lemma A.3, if the other player accepts new hot potatoes, then accepting new hot potatoes is a best response for a player if and only if $\rho \leq \rho^2$; and by Lemma A.4, if the other player rejects new hot potatoes, then rejecting new hot potatoes is a best response for a player if and only if $\rho \geq \rho^1$. So, if $\rho < \rho^1$, there is a unique Markov perfect equilibrium, and in this equilibrium, all players accept new hot potatoes; likewise, if $\rho > \rho^2$, there is a unique Markov perfect equilibrium, and in this equilibrium, players reject new hot potatoes. For the intermediate case $\rho \in [\rho^1, \rho^2]$, there are two pure Markov perfect equilibria, one in which players reject new hot potatoes, and one in which they reject them. By Lemma A.5, there can additionally be mixed Markov perfect equilibria for these parameters: If $\rho \in (\rho^1, \rho^2)$, there is a unique Markov perfect equilibrium in strictly mixed strategies. In this equilibrium, both players accept new hot potatoes with probability

$$q = \frac{u_h\left(1 - \frac{1}{2}\beta\gamma\right) + \frac{1}{4}\beta\gamma u_s - \frac{1}{8}\beta^2\gamma u_s}{\frac{1}{4}\beta(1-\gamma) u_h},$$

which lies strictly between 0 and 1. If $\rho = \rho^1$ or $\rho = \rho^2$, there is a continuum of mixed Markov perfect equilibria. In these equilibria, one player accepts or rejects new hot potatoes with probability one, whereas the other accepts new hot potatoes with some probability $q \in (0,1)$. □

A.3 Proof of Proposition 3.2

By Proposition A.2 in Appendix A.1, we can restrict attention to policies in reduced strategies. For ease of exposition, we will drop the qualifier "reduced" in this proof when referring to reduced policies or reduced strategies.

We characterize the optimal policy σ^*. By standard arguments, we can restrict attention to pure stationary Markov policies (Puterman, 2014, Thm. 5.5.3, Prop. 6.2.1). That is, we can restrict attention to policies $\sigma = (\sigma_1, \sigma_2)$, with $\sigma_i : T \to \{0, 1\}$ for $i \in \{1, 2\}$, where $\sigma_i(\tau) = 1$ (resp. $\sigma_i(\tau) = 0$) indicates that player i accepts (resp. rejects) a task of type τ (conditional on being matched).

The transition probabilities $(p_\theta^{\theta'})_{\theta,\theta' \in \Theta}$ induced by a policy σ are as in the proof of Proposition 3.1 (with the minor notational change that $\sigma_i(\tau)$ replaces q_i^τ in the relevant expressions for a policy σ).

The proof follows from a series of lemmas. The first two lemmas show that under the optimal policy, players accept old sweet potatoes but not old hot potatoes.

Lemma A.6. *For all $i \in \{1, 2\}$, $\sigma_i^{**}(o, s) = 1$.*

Proof. Immediate. Accepting an old sweet potato gives a positive payoff $u_s > 0$ in the current period and does not influence the transition probabilities (i.e., $p_\theta^{\theta'}$ is independent of $\sigma_i^{**}(o, s)$ for all $i \in \{1, 2\}$, $\theta, \theta' \in \Theta$). $\square$

Lemma A.7. *For all $i \in \{1, 2\}$, $\sigma_i^{**}(o, h) = 0$.*

Proof. Immediate. Accepting an old hot potato gives a negative payoff $u_h < 0$ in the current period and does not influence the transition probabilities (i.e., $p_\theta^{\theta'}$ is independent of $\sigma_i^{**}(o, h)$ for all $i \in \{1, 2\}$, $\theta, \theta' \in \Theta$). $\square$

Lemmas A.6–A.7 give the following Bellman equations:[11]

$$\begin{aligned}
V_{\sigma^{**}}(S) &= \tfrac{1}{2}\left(\sigma_1^{**}(n,s)+\sigma_2^{**}(n,s)\right)u_s + \\
&\qquad \beta\left[p_S^S V_{\sigma^{**}}(S) + p_S^H V_{\sigma^{**}}(H) + p_S^{SS} V_{\sigma^{**}}(SS) + p_S^{HS} V_{\sigma^{**}}(HS)\right];\\
V_{\sigma^{**}}(H) &= \tfrac{1}{2}\left(\sigma_1^{**}(n,h)+\sigma_2^{**}(n,h)\right)u_h + \\
&\qquad \beta\left[p_H^S V_{\sigma^{**}}(S) + p_H^H V_{\sigma^{**}}(H) + p_H^{SH} V_{\sigma^{**}}(SH) + p_S^{HH} V_{\sigma^{**}}(HH)\right];\\
V_i(SS) &= \tfrac{1}{2}u_s + \tfrac{1}{2}V_{\sigma^{**}}(S) + \tfrac{1}{2}\beta\left[\gamma V_{\sigma^{**}}(SS) + (1-\gamma)V_{\sigma^{**}}(HS)\right];\\
V_{\sigma^{**}}(SH) &= V_{\sigma^{**}} - \tfrac{1}{2}u_s;\\
V_{\sigma^{**}}(HH) &= \tfrac{1}{2}V_{\sigma^{**}}(H) + \tfrac{1}{2}\beta\left[\gamma V_{\sigma^{**}}(SH) + (1-\gamma)V_{\sigma^{**}}(HH)\right];\\
V_{\sigma^{**}}(SH) &= V_{\sigma^{**}} + \tfrac{1}{2}u_s;
\end{aligned}$$

Lemma A.8. *For all $i \in \{1,2\}$, $\sigma_i^{**}(n,s) = 1$.*

Proof. The proof is similar to (but distinct from) the proof for Lemma A.2. We need to show that for any player that is matched with a new sweet potato, the value of accepting the sweet potato is greater than the value of rejecting it.

So, suppose that a player is matched with a new sweet potato. There are three possibilities: either both players accept new sweet potatoes, or one player accepts new sweet potatoes (and the other one does not), or no player accepts sweet potatoes. Hence, we need to show that

$$\begin{aligned}
u_s + \beta\left[\gamma V_{\sigma^{**}}(S) + (1-\gamma)V_{\sigma^{**}}(H)\right] &> \beta\left[\gamma V_{\sigma^{**}}(SS) + (1-\gamma)V_{\sigma^{**}}(HS)\right]; \text{and}\\
u_s + \beta\left[\gamma V_{\sigma^{**}}(S) + (1-\gamma)V_{\sigma^{**}}(H)\right] &> \tfrac{1}{2}u_s + \tfrac{1}{2}\beta\left[\gamma V_{\sigma^{**}}(S) + (1-\gamma)V_{\sigma^{**}}(H)\right] + \\
&\qquad \tfrac{1}{2}\beta\left[\gamma V_{\sigma^{**}}(SS) + (1-\gamma)V_{\sigma^{**}}(HS)\right];
\end{aligned}$$

where the first inequality says that, conditional on a player being matched with a new sweet potato, the expected discounted sum of aggregate payoffs when both players accept new sweet potatoes exceeds the expected discounted sum of aggregate payoffs when no player accepts new sweet potatoes; and the second inequality says that the expected discounted sum of aggregate payoffs when both players accept new sweet potatoes exceeds the expected discounted sum of aggregate payoffs when only one player accepts new sweet potatoes. It thus suffices to show that, for any $s < 1$,

$$\begin{aligned}
u_s + \beta\left[\gamma V_{\sigma^{**}}(S) + (1-\gamma)V_{\sigma^{**}}(H)\right] > s u_s + s\beta\left[\gamma V_{\sigma^{**}}(S) + (1-\gamma)V_{\sigma^{**}}(H)\right] + \\
(1-s)\beta\left[\gamma V_{\sigma^{**}}(SS) + (1-\gamma)V_{\sigma^{**}}(HS)\right],
\end{aligned}$$

[11]Note that the expression for $V_{\sigma^{**}}(\theta)$ for $\theta \in \Theta$ is similar to, but distinct from, the continuation payoff $V_i(\theta)$ for a player i in the proof of Proposition 3.1 (with $q_i^{(o,s)} = 1$ and $q_i^{(o,h)} = 0$ for each player i).

which is equivalent to showing that

$$u_s + \beta\left[\gamma V_{\sigma^{**}}(S) + (1-\gamma)V_{\sigma^{**}}(H)\right] > \beta\left[\gamma V_{\sigma^{**}}(SS) + (1-\gamma)V_{\sigma^{**}}(HS)\right]. \tag{A.13}$$

As in the proof of Lemma A.2, a policy under which players accept only sweet potatoes (whether old or new) can guarantee a positive payoff whenever a player is matched to a sweet potato. So, under the optimal policy σ^*,

$$\max_{s\in\{0,\frac{1}{2},1\}} \left\{sh + \beta s\left[\gamma V_{\sigma^{**}}(S) + (1-\gamma)V_{\sigma^{**}}(H)\right] + \beta(1-s)\left[\gamma V_{\sigma^{**}}(SS) + (1-\gamma)V_{\sigma^{**}}(HS)\right]\right\} > 0 \tag{A.14}$$

First suppose that $\gamma V_{\sigma^{**}}(SS) + (1-\gamma)V_{\sigma^{**}}(HS) < 0$. Then, by (A.14), $s \neq 0$ under σ^{**}. So, it remains to consider $s \in \{\frac{1}{2}, 1\}$. For $s = \frac{1}{2}$, we have

$$\begin{aligned}\tfrac{1}{2}u_s + \tfrac{1}{2}\beta\left[\gamma V_{\sigma^{**}}(S) + (1-\gamma)V_{\sigma^{**}}(H)\right] + \tfrac{1}{2}\beta\left[\gamma V_{\sigma^{**}}(SS) + (1-\gamma)V_{\sigma^{**}}(HS)\right] <\\ \tfrac{1}{2}u_s + \tfrac{1}{2}\beta\left[\gamma V_{\sigma^{**}}(S) + (1-\gamma)V_{\sigma^{**}}(H)\right].\end{aligned} \tag{A.15}$$

So, if

$$\tfrac{1}{2}u_s + \tfrac{1}{2}\beta\left[\gamma V_{\sigma^{**}}(S) + (1-\gamma)V_{\sigma^{**}}(H)\right] + \tfrac{1}{2}\beta\left[\gamma V_{\sigma^{**}}(SS) + (1-\gamma)V_{\sigma^{**}}(HS)\right] > 0, \tag{A.16}$$

then the right-hand side of (A.15) is positive, and it follows that

$$\begin{aligned}\tfrac{1}{2}u_s + \tfrac{1}{2}\beta\left[\gamma V_{\sigma^{**}}(S) + (1-\gamma)V_{\sigma^{**}}(H)\right] + \tfrac{1}{2}\beta\left[\gamma V_{\sigma^{**}}(SS) + (1-\gamma)V_{\sigma^{**}}(HS)\right] <\\ \tfrac{1}{2}u_s + \tfrac{1}{2}\beta\left[\gamma V_{\sigma^{**}}(S) + (1-\gamma)V_{\sigma^{**}}(H)\right] < u_s + \beta\left[\gamma V_{\sigma^{**}}(S) + (1-\gamma)V_{\sigma^{**}}(H)\right].\end{aligned}$$

That is, if (A.16) holds, $s \neq \frac{1}{2}$ under σ^{**}, so we conclude that $s = 1$ under σ^{**}. So suppose (A.16) is not satisfied. Then it follows directly from (A.14) that $s = 1$ under σ^{**}.

Next suppose $\gamma V_{\sigma^{**}}(SS) + (1-\gamma)V_{\sigma^{**}}(HS) \geq 0$. From the Bellman equations,

$$\begin{aligned}\left(\gamma V_{\sigma^{**}}(SS) + (1-\gamma)V_{\sigma^{**}}(HS)\right)\left(1 - \tfrac{1}{2}\beta\right) =\\ \tfrac{1}{2}u_s(1 - \tfrac{1}{2}\beta(1-\gamma)) + \tfrac{1}{2}\left[\gamma V_{\sigma^{**}}(S) + (1-\gamma)V_{\sigma^{**}}(H)\right].\end{aligned}$$

Therefore,

$$\begin{aligned}\left(\gamma V_{\sigma^{**}}(S) + (1-\gamma)V_{\sigma^{**}}(H)\right) - \left(\gamma V_{\sigma^{**}}(SS) + (1-\gamma)V_{\sigma^{**}}(HS)\right) =\\ \left[\gamma V_{\sigma^{**}}(SS) + (1-\gamma)V_{\sigma^{**}}(HS)\right](1-\beta) - \tfrac{1}{2}u_s(1 - \tfrac{1}{2}\beta(1-\gamma)).\end{aligned}$$

It is then easy to check that (A.13) holds, proving the claim. $\square$

We are now ready to prove Proposition 3.2. By Lemmas A.6–A.8, under the optimal matching protocol, players accept sweet potatoes (old or new) and reject old hot potatoes. It thus remains to identify the conditions under which players accept new hot potatoes. The discounted sum of aggregate payoffs when players accept sweet potatoes and new hot potatoes (but reject old hot potatoes) equals

$$V_{accept}(u_s, u_h, \beta, \gamma) := \frac{\gamma u_s + (1-\gamma)u_h}{1-\beta}. \tag{A.17}$$

We can compare this to the discounted sum $V_{reject}(u_s, u_h, \beta, \gamma)$ of aggregate payoffs when players accept sweet potatoes but reject hot potatoes (old or new), which can be calculated from the Bellman equations. We have

$$V_{reject}(u_s, u_h, \beta, \gamma) = \gamma V_\sigma(S) + (1-\gamma)V_\sigma(H),$$

where σ is the policy under which players accept sweet potatoes but reject hot potatoes (i.e., for all $i \in \{1,2\}$, $\sigma_i(o,s) = \sigma_i(n,s) = 1$, $\sigma_i(o,h) = \sigma_i(n,h) = 0$). From the Bellman equations, we have

$$\begin{aligned}(1-\beta\gamma)\left[\gamma V_\sigma(S) + (1-\gamma)V_\sigma(H)\right] &= \gamma u_s + \beta(1-\gamma)[\gamma V_\sigma(SH) + (1-\gamma)V_\sigma(HH)];\\ (1-\tfrac{1}{2}\beta)\left[\gamma V_\sigma(SH) + (1-\gamma)V_\sigma(HH)\right] &= \tfrac{1}{4}\beta\gamma u_s + \tfrac{1}{2}[\gamma V_\sigma(S) + (1-\gamma)V_\sigma(H)].\end{aligned}$$

Solving this system of two linear equations for the two unknown terms $[\gamma V_\sigma(S) + (1-\gamma)V_\sigma(H)]$ and $[\gamma V_\sigma(SH) + (1-\gamma)V_\sigma(HH)]$ yields

$$V_{reject}(u_s, u_h, \beta, \gamma) = \frac{\frac{1}{2}\gamma u_s\left(\beta^2(1-\gamma)^2 + 2\,(2-\beta)\right)}{-\gamma\beta(1-\beta) + 2\,(1-\beta)}. \tag{A.18}$$

Combining (A.17) and (A.18) and using that $\rho = |u_h/u_s|$, we find that $V_{eat}(u_s, u_h, \beta, \gamma) \geq V_{reject}(u_s, u_h, \beta, \gamma)$ if and only if

$$\rho \leq \frac{\frac{1}{4}\beta\gamma\,(2-\beta\,(1-\gamma)}{1-\frac{1}{2}\beta\gamma}.$$

The proof then follows by setting $\rho^* := \frac{\frac{1}{4}\beta\gamma\,(2-\beta\,(1-\gamma)}{1-\frac{1}{2}\beta\gamma}$. □

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